The Inevitable Tension between Long-term and Short-term Managerial and Investor Incentives

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ABSTRACT

The paper considers a model in which (1) managers allocate effort to both short- and long-term projects, and (2) there is feedback between the managerial incentive contract and the number of speculators collecting information on each type of project. More weight placed on near-term price results in more speculation based on information about the short-term project, which induces further increases in the weight placed on near-term price. This feedback effect can result in short-term speculation crowding out the collection of long-term information, which in turn results in the withdrawal of incentives aimed at inducing effort in more profitable long-term projects. The paper shows that the equilibrium that obtains depends upon adjustment costs and initial conditions and is, in general, not efficient. Such outcomes are consistent with concerns about managerial and investor short-termism recently expressed by policy makers and market participants (e.g., the Aspen Institute). The paper considers the efficacy of various corporate and public policy remedies.

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I. Introduction

Among the many forces that contributed to the recent financial crisis, many policy makers, market participants, and market observers blame managerial short-termism and the prevalence of managerial incentives that focus too much on near-term stock price rather than long-term value; presumably, by focusing on their firm’s current stock price, managers neglect risks that threaten long-term solvency and stability. This argument clearly views near-term price as not adequately reflecting long-run value; if it did, then management would take actions to mitigate long-run vulnerabilities in an effort to boost near-term price (i.e., by caring about the near-term stock price, managers care about long-term value). As such, many of the discussions about the financial crisis also blame investor short-termism for not making current prices reflect long-term risks. The argument is that, although equity prices are supposed to impound available information about the long-term viability of the firm, since the majority of investment decisions (mostly made by professional portfolio managers

1For example, in an October 2009 article in the Atlantic, former Senior Vice President for Law and Public Affairs at General Electric and current senior fellow at Harvard’s Kennedy School of Government and Harvard Law School’s Program on Corporate Governance Ben W. Heineman Jr. stated: “As we now know all too well, the credit crisis and the global recession stemmed, in important part, from stark failures of boards of directors and operating business leadership in important financial institutions: the witchs brew of leverage, poor risk management, creation of toxic products, lack of liquidity all made more poisonous by compensation systems which rewarded short-term revenues/profits without regard to risk.” In remarks to the National Press Club in July of 2011, former Chairman of the Federal Deposit Insurance Corporation (FDIC) Sheila Bair concurred: “The compensation of loan officers, portfolio managers and bank CEOs was typically based on current-year loan volume, earnings or stock price, with little regard for the risks that were building up in the system” (italics added).

2For example, a committee of the Aspen Institute, consisting of 30 current or retired CEOs, legal experts, and government officials (including such notable members as Berkshire Hathaway CEO Warren E. Buffett, Vanguard founder John C. Bogle, and TIAA-CREF President and CEO Roger W. Ferguson, Jr.), signed a statement decrying short-termism, proclaiming that “[...fund managers with a primary focus on short-term trading gains have little reason to care about long-term corporate performance or externalities, and so are unlikely to exercise a positive role in promoting corporate policies, including appropriate proxy voting and corporate governance policies, that are beneficial and sustainable in the long-term.” This position is also echoed by UCLA Law Professor Lynn Stout (also a member of the Aspen Institute committee) in a Wall Street Journal blog: “The pursuit of short-term trading profits also has a corrosive effect on the wider economy because it distracts corporate managers who must respond to short-term investors demands from the important business of planning for and investing in the future. [...]Corporations build railways, specialized manufacturing facilities, trusted international brand names, mass-produced software, new drugs and medical devices. Yet, it can be difficult or impossible for corporate directors and executives to focus their attention on such projects when they are constantly being called upon instead to meet quarterly earnings targets and to raise tomorrows stock price.”
who are gauged by their short-term performance) are driven by short-term gain, very little information about the long-term performance of the firm is collected, resulting in near-term stock prices that contain little information about the long run. Thus, when incentive contracts are written on near-term equity values, managers have little incentive to take actions that improve long-run value.

These arguments have led many to suggest or impose that incentive contracts place more weight on longer-term returns/value. For example, in what amounts to a reallocation of contractually specified weight from short- to long-term performance metrics, the FDIC recently adopted a rule that “allows the agency to claw back two years’ worth of compensation from senior executives and managers responsible for the collapse of a systemic, non-back financial firm...” (Bair (2011)). Also, New York Times banking reporter Eric Dash noted “[o]ne idea is for ‘clawbacks’ to be strengthened to include bad performance, not just wrongdoing or fraud.” Further, he suggests “...it might be worthwhile to consider withholding a bigger portion of trader’s bonus over a more extended period of time, perhaps several years or more, in an escrow account. That way, it could be adjusted up or down based on the trader’s actual results....” In addition, others have suggested investors be “encouraged” to take a longer-term view too. For example, the Aspen Institute committee (see footnote 2) suggests three policies: (1) “Revise capital gains tax provisions or implement an excise tax in ways that are designed to discourage excessive share trading and encourage long-term share ownership....” (2) “Remove limitations on capital loss deductibility for very long-term holdings....” (3) “In exchange for enhancing shareholder participation rights, consider adopting minimum holding periods or time-based vesting....”

Although statements/beliefs such as those identified above appear to be fairly common following the financial crisis, these beliefs raise a whole raft of questions - many of which don’t have obvious answers in the academic literature. One question is whether (and why) there is an excessive incentive to collect information on short-term prospects - especially if, as is presumed in many of the statements above, long-term prospects are more important to
long-run value. Presumably, any investor that can uncover a lack of long-term viability would seem to be able to profit in the short run by publicizing such inadequacies. Is there some sense in which short-term information “crowds out” more important long-term information? In addition, it is also unclear how putting greater weight on far-term value would affect the incentives to collect information on short- versus long-term projects. Indeed, prices are supposed to transport future values to the present; anything that improves long-term value (and the manager’s compensation based on long-term value) should also improve near-term value (and the manager’s compensation based on near-term value). Moreover, implicit in recommendations for centralized regulation (e.g., as in the FDIC’s clawback provisions and the Aspen Institute Committee’s suggested revisions to capital gains taxation) is the belief that there are externalities that private contracting cannot solve; what is the nature of these externalities (especially in terms of contracting)?

In this paper, we develop a simple model to investigate the issues raised above. Specifically, the paper develops an agency model in which a manager allocates (personally costly) effort to both a short- and a long-term project, both of which positively affect the long-run value of the firm. In order to induce effort, the manager is compensated according to an incentive contract that specifies pay contingent on both near- and far-term stock prices. Hereafter, we refer to the sensitivities of the manager’s compensation to the near-term price and far-term price as, respectively, the near-term weight and the far-term weight. The paper also considers the incentives of speculators to collect and trade on information on each of the projects. Depending upon speculators ability to profitably trade on such information, the amount of each type of information collected in the first place and the amount of that information that is reflected in the near-term stock price may vary by project type. In order to determine whether the types of distortions described in the quotes above are possible, we examine how the initial, near-term, and long-term values of the firm depends upon the

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3 This question was explicitly asked in Heineman (2009): “Indeed, important questions have been raised about the role institutional investors played in causing the melt-down by pressuring financial service entities to take undue risk for short-term profits. Did the short-term investors crowd out long term investors in influencing corporations and, if so, is this likely to be the future pattern?” (italics added.)
incentive contract, the characteristics of the equity market, and the market for information.

The model generates four main results. First, when the far-term value of the firm contains greater uncertainty independent of the manager’s current effort, it is optimal to place compensation weights on both near-term and far-term equity values. The far-term firm value fully reflects the impact of the manager’s short-term and long-term effort choices, while the near-term price only partially reflects speculators’ (potentially noisy) information about the manager’s effort choices. As a result, far-term weight is more effective than near-term weight at inducing the manager to exert effort on both the short- and long-term projects. But, since the far-term value depends upon more things beyond the manager’s control at the time the manager exerts effort, weight placed on the far-term value generates more risk to the manager. Since the manager must be compensated for this risk via a higher average wage, more weight placed on far-term value is more expensive to shareholders. Thus, shareholders opt to place weight on both the near-term and the far-term firm value in order to reduce compensation costs.

Second, the nature of the equity and information markets affects the relative ability of the near-term price to induce effort on each type of project – independently of the productivity of each type of effort on long-run outcomes. In particular, when the number of speculators collecting short-term information is higher than the number collecting long-term information, then the near-term price will reflect relatively more of the short-term information, making the near-term price more effective (and less costly) at inducing effort on the short-term project. As a result, if there are more speculators collecting information on the short-term project, the market will “encourage” the short-term project relative to the long-term project.

The third result is that more weight placed on the near-term price increases the expected trading profits of the speculators collecting and trading on information about the short-term project. Further, as the number of speculators collecting information on the short-term project increases with the increase in expected profits, the near-term price reflects more short-term information, which in turn makes shareholders want to increase the weight on
the near-term price. This happens because when more weight is placed on the near-term price
the manager exerts relatively more effort on the short-term project and, as a result, a larger
component of the firm’s future value is determined by the short-term project. This then
leads to an increase in the incentive for speculators to collect information on the short-term
project. And, as more speculators collect (and compete) on information about the short-
term project, incentives based on the near-term stock price become cheaper (as the prices
have less noise related to liquidity trading), which encourages shareholders to place more
weight on the near-term stock price. As a consequence, there exist equilibria in which more
weight is placed on the near-term price and the manager exerts more effort on the short-term
project, even when managerial effort is more productive if exerted on the long-term project.

The fourth result is that trade based on information on the short-term project crowds
out the collection of information about the long-term project. This is true because the
future value of the firm depends more on the manager’s effort and productivity in the short-
term project, which is less correlated with information on the long-term project collected
by long-term informed traders. Thus, the market sustains fewer informed traders collecting
information on the long-term project.

In addition to the theoretical literature that shows that there can be a divergence between
the private incentive to collect information and its social value (e.g., Hirshleifer (1971),
Marshall (1974) and Hakansson, Kulkel, and Olson (1982), the literature has developed
models that specifically imply a trade-off between short- and long-term projects. For
example, the signal jamming models in Narayanan (1985) and Stein (1989) show that, even
though no one is fooled, managers may attempt to manipulate shareholders’ beliefs about
long-run value by boosting short-term earnings via excessive borrowing from the future. Peng
and Roell (2012) develop a model for optimal executive compensation where managers are
prone to manipulate and their propensity to manipulate is random. The resulting equilibrium
contract weights (written on near-term stock price and far-term value) depend on the degree
of manipulation uncertainty: when manipulation uncertainty is high, more weight is placed
on near-term stock price. In addition, Bolton, Scheinkman, and Xiong (2009) provide a model in which the market, by assumption, generates a near-term price that provides a poor signal of long-term value. In their model, the equilibrium incentive contract places weight on both near-term price and long-term value. As a result, the managers make sub-optimal decisions relative to situations in which the market price is a better signal of long-term value. These models rely on the existence of uncertainty about the information content of market prices with respect to long-term firm value. These elements are not present in the model we develop herein.

A closely related paper is Paul (1992), which also considers the efficacy of equity-based contracts when the manager must allocate effort across multiple projects. A key difference between our model and Paul (1992) is that in Paul (1992) the productivity of the manager is known by the market and the parameters of the compensation contract do not affect the speculators’ incentive to collect information. Furthermore, in Paul (1992), the allocation of information across projects is taken as given. In our model, informed traders endogenously choose the type of information to obtain given the manner in which the contract affects the incentive to collect specific type of information.

The remainder of the paper is organized as follows. Section I describes the model. Section II characterizes the equilibrium in the securities market taking the contract as given. This section derives (1) the optimal amounts of effort to exert on the short- and long-term projects given the contract and the price function, (2) the optimal trades of speculator as a function of the information they collect on short- and/or long-term projects and the parameters of the price function, (3) the unconditional expected speculative profits as a function of the number of speculator collecting each type of signal and the parameter of the managerial incentive contract, and (4) the equilibrium price function. Section III discusses the setting of the parameters of the incentive contract and the overall equilibrium. This section also shows the nature of the contracting inefficiency. Section IV considers potential remedies. Section V concludes.
II. The Model

A. Managerial Effort and Long-run Value

The principal-agent model runs over three periods \((t = 0, 1, \text{ and } 2)\) as depicted in the timeline in Table I. At \(t = 2\), the terminal value of the firm \(v\) is distributed to claimants (which include the shareholders and the manager). The terminal value of the firm is simply the sum of the values from a short-term project \((v_S)\) and a long-term project \((v_L)\), both of which are initiated at \(t = 0\):

\[
v = v_S + v_L \quad (1)
\]

The values of the projects (realized at \(t = 2\)) depend upon the amount of effort (denoted \(e_S\) and \(e_L\)) the manager (i.e., the agent) exerts on each of these projects at \(t = 0\). Specifically, the realized value of the short-term project is

\[
v_S = \gamma_S e_S + \eta_S, \quad (2)
\]

where \(\gamma_S\) is the marginal productivity of the manager’s short-term effort \(e_S\) and \(\eta_S\) is a short-run random component beyond the manager’s control, where \(\eta_S \sim N(0, \sigma_{\eta_S}^2)\). The marginal productivity of effort with respect to the short-term project \(\gamma_S\) is, with respect to the markets information set, random, with \(\gamma_S^2 \sim N(\bar{\gamma}_S^2, \sigma_{\gamma_S}^2)\). However, the realized value of \(\gamma_S\) (or \(\gamma_S^2\)) is known by the manager at the time the manager exerts effort on the short-term project.

The realized value of the long-term project is

\[
v_L = \gamma_L e_L + \eta_L + \eta_T, \quad (3)
\]

\footnote{As will be obvious later, we assume that the square of the marginal productivity (rather than the level of the marginal productivity itself) is normally distributed in order to obtain closed-form solutions for various endogenous variables.}
where $\gamma_L$ is the marginal productivity of long-term effort $e_L$, $\eta_L$ is a short-run random component with respect to the long-term project that is beyond the manager’s control and is uncorrelated with $\eta_S$ (i.e., $E(\eta_S \eta_L) = 0$), and $\eta_T$ is a long-run random component that affects the outcome of the long-term project that is also beyond the manager’s control and is independent of both $\eta_S$ and $\eta_L$. Both $\eta_L$ and $\eta_T$ are normally distributed; $\eta_L \sim N(0, \sigma^2_{\eta_L})$ and $\eta_T \sim N(0, \sigma^2_{\eta_T})$. As with the short-term project, the marginal productivity of effort with respect to the long-term project $\gamma_L$ is, with respect to the market’s information set, random, with $\gamma^2_L \sim N(\bar{\gamma}_L^2, \sigma^2_{\gamma_L})$. Again, the realized value of $\gamma_L$ (or $\gamma^2_L$) is known by the manager at the time the manager exerts effort on the long-term project.

One of the main differences between the short- and long-term projects is that, since the long-term project is realized farther in the future, its realization depends upon more things beyond the manager’s control at $t = 0$ (when effort is exerted). That is, the far-term payoff to the long-term projects includes $\eta_T$ (with $\sigma^2_{\eta_T} > 0$) while no such random component exists for the long-term payoff of the short-term project. As is discussed below, the timing of the realization and availability of information on each of these components is such that $\sigma^2_{\eta_T} > 0$ is not equivalent to $\sigma^2_{\eta_L} > \sigma^2_{\eta_S}$ with $\sigma^2_{\eta_T} = 0$. In addition, in keeping with the presumption that long-term projects lead to greater long-run value than short-term projects, we assume that the expected managerial productivity of effort is greater when exerted on the long-term rather than the short-term project: $\bar{\gamma}^2_L > \bar{\gamma}^2_S$.

B. Speculators’ Information on Projects

An important part of the contracting environment is the amount of information about the manager’s effort exerted on each type of project that will be reflected in the near-term price

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5 For example, a long-term project may be to develop new markets in developing countries while a short-term project might entail investing in a new technology that reduces production costs. While the impact of the cost saving technology will become apparent in the near term, whether the new market turns out to be lucrative or not depends upon both the actions and efforts of the management but also on many factors beyond the managers control such as political reform that might liberalize or restrict foreign investment.

6 All of the expressions derived herein are general and do not rely on a particular ranking of the various parameters. We merely maintain certain ranking assumptions in order to simplify the exposition and relate the analysis to the claims found in the business press.
and the terminal value. The incentive to collect and trade on such information depends upon the informational efficiency of the market for the firm’s shares, which we assume is open at $t = 1$. Investors (or speculators) in this market are risk neutral and value the stock at the expected value of the terminal value conditional on any information that is available at $t = 1$. The information that is available at $t = 1$ includes both a costly noisy signal $\theta_S$ of the value of the short-term project and a costly noisy signal $\theta_L$ of the short-term component of the value of the long-term project $\gamma_L e_L + \eta_L$. Specifically, the signals are related to project values as follows:

$$\theta_S = \gamma_S e_S + \eta_S + \varepsilon_S,$$

$$\theta_L = \gamma_L e_L + \eta_L + \varepsilon_L$$

where $\varepsilon_i \sim N(0, \sigma^2_{\varepsilon_i})$ and $E[\varepsilon_i \eta_j] = E[\varepsilon_i \eta_T] = 0$ for $i, j \in S, L$. Since the signal $\theta_S$ is based on information about short-run outcomes that may have already come to fruition while $\theta_L$ is with respect to outcomes that are farther in the future, then it is likely that the long-term signal is noisier (i.e., $\sigma^2_{\varepsilon_L} > \sigma^2_{\varepsilon_S}$) and/or more costly (i.e., $C_L \geq C_S$, where $C_i$ denotes the cost of obtaining an $i$-type signal, for $i \in S, L$). We will refer to any agent that obtains $\theta_S$ as a short-term informed investor; any agent that obtains $\theta_L$ is a long-term informed investor. By obtaining signals $\theta_S$ and/or $\theta_L$, traders obtain information on $\eta_S$ and/or $\eta_L$; it is assumed that signals that provide information on $\eta_T$ are not available (for any cost) at $t = 0$ or $t = 1$. Thus, $\sigma^2_{\eta_T} > 0$ but $\sigma^2_{\eta_L} = \sigma^2_{\eta_S}$ is not equivalent to $\sigma^2_{\eta_L} > \sigma^2_{\eta_S}$ with $\sigma^2_{\eta_T} = 0$.\footnote{If the investors were risk averse, then the price would be the expected terminal value minus a risk premium. If the market participants know the parameters of the model, then the risk premium will be a non-random constant with respect to investors’ information sets. As a result, for a given amount of information collected, the information content of the price in a risk-averse economy will be the same as in a risk-neutral economy. However, since in a risk-averse economy the amount of information collected may depend upon the risk aversion of investors, the amount of information collected may be different in a risk-averse versus a risk-neutral economy.}

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C. Market Prices

Initially (i.e., at $t = 0$), the price of the stock is the unconditional expected terminal value: $p_0 = E[v']$, where $v'$ denotes the terminal value net of the manager’s compensation. The stock price at $t = 1$ is this unconditional expected terminal value plus any deviation created by the information content of investors trading at $t = 1$. That is, the price $p$ in the secondary market at $t = 1$ is the expected terminal value conditional on the net order flow given informed and liquidity trades:

$$p = E[v' | \Psi] = p_0 + \lambda \Psi$$

where $\Psi \equiv \sum_{m=1}^{N_1^L} X_m^S + \sum_{m=1}^{N_1^L} X_m^L + Z$ is the net order flow at $t = 1$, $Z$ denotes the trades of liquidity traders (with $Z \sim N(0, \sigma_Z^2)$), $X_m^S$ is the demand of the $m$th short-term informed investor (of which there are $N_1^S$), and $X_m^L$ is the demand of the $m$th long-term informed investor (of which there are $N_1^L$). With competitive market makers who make zero expected profits, $\lambda = \frac{\text{cov}(\Psi, v')}{\text{var}(\Psi)}$.

To the extent that liquidity traders may have some discretion as to which securities to trade to satisfy their liquidity needs, the variance of liquidity trades $\sigma_Z^2$ for a particular firm may depend upon cross-sectional variation in both endogenous and exogenous characteristics of firms traded in the market. For now, however, we take the value of $\sigma_Z^2$ as a parameter and derive results conditional upon its value. After these conditional results are obtained, the forces that determine how liquidity trades will be distributed (and their impact on the over-all equilibrium) will be discussed.

D. Incentive Compensation and the Manager’s Objective Function

At $t = 1$, the manager is (partially) compensated based on a short-term measure of performance. Specifically, the compensation at $t = 1$ is $\omega_p p$, where “performance” is
measured by the near-term price $p$ and $\omega_p$ is the short-term pay-for-performance sensitivity. The manager will also receive compensation in the long-term (at $t = 2$) based on the long-term value of the firm realized at $t = 2$. In particular, at $t = 2$, the manager receives compensation $\omega_v v$, where $v$ is the realized far-term value of the firm gross of managerial compensation and $\omega_v$ is the long-run pay-for-performance sensitivity. At $t = 0$, the manager is paid a fixed wage $\omega_0$. Thus, the manager’s compensation contract is characterized as an ordered triple $\omega = (\omega_0, \omega_p, \omega_v)$. To simply focus on the incentives and the risks associated with these incentives, we assume that the manager has no time preference; that is, the objective function of the manager is based on the simple sum of his compensation payments:

$$w = \omega_0 + \omega_p p + \omega_v v$$  \hspace{1cm} (7)

That is, even though the payments $\omega_0$, $\omega_p p$, and $\omega_v v$ are made at different times, they all receive equal weight; thus, the timing of the compensation payments is unimportant. Under this assumption, there is no benefit to paying the manager early; in actual markets, one reason to base the manager’s pay on near-term market price and/or performance is that the manager faces liquidity constraints and capital market imperfections that prevent him/her from being able to borrow against expected future performance in order to consume during earlier periods. This motivation for placing weight on the near-term value does not exist under the assumptions made herein. Thus, if there is an incentive to put weight on near term price here, then there will be even stronger incentives (based on liquidity) in actual markets. The issue here is simply whether there is a justification for placing weight to near-term price based on incentives independent of any liquidity justification.

Although the manager has no time preference, s/he is risk averse and has disutility to both short- and long-run effort. Specifically, the manager has a negative exponential utility.
with a constant absolute risk averse (CARA) risk preference given by

\[ u(w, e_S, e_L) = \exp \left[ -\gamma \left( w - \sum_{i \in (S, L)} \psi(e_i) \right) \right], \tag{8} \]

where \( \gamma \) is the manager’s coefficient of constant absolute risk aversion (i.e., \( \gamma = -\frac{w''}{w} > 0 \)), and the disutility function \( \psi(e_i) \) is given by

\[ \psi(e_i) = \frac{\delta_i}{2} e_i^2 \tag{9} \]

for \( i \in S, L \). Given effort levels \( e_S \) and \( e_L \) and compensation \( w \), the manager’s expected utility satisfies

\[ E[u(w, e_S, e_L)] \propto E[w] - \frac{\gamma}{2} \text{var}(w) - \frac{\delta_S}{2} e_S^2 - \frac{\delta_L}{2} e_L^2. \tag{10} \]

The manager is willing to accept any contract that produces an expected utility greater than or equal to his reservation utility, which, without a loss of generality, we assume is zero.

Since we allow for the manager’s marginal productivity with respect to each type of project to differ (i.e., \( \gamma_S \) need not equal \( \gamma_L \)), there is no need to have the disutility of effort exerted differ by the type of project; thus, we can have \( \delta_i = \delta \) for \( i \in S, L \).

We further assume that no one in the model has time preference; thus, the net interest rate is zero.

\textit{E. The Shareholders’ Problem}

Prior to the manager exerting effort at \( t = 0 \), the board of directors, acting on behalf of long-term shareholders (i.e., shareholders that will hold the equity in the firm until \( t = 2 \)),
picks the contract $\omega = (\omega_0, \omega_p, \omega_v)$ to solve the following problem:

$$
\max_{\omega_0, \omega_p, \omega_v} E[v(\hat{e}_S, \hat{e}_L)] - E[w(\hat{e}_S, \hat{e}_L)]
$$

subject to (PC) $E[w|\hat{e}_S, \hat{e}_L] - \frac{\gamma}{2}\text{var}(w|\hat{e}_S, \hat{e}_L) - \frac{\delta_S}{2}(\hat{e}_S)^2 - \frac{\delta_L}{2}(\hat{e}_L)^2 = 0$

and (IC) $\hat{e}_S, \hat{e}_L \in \arg\max E[w|e_S, e_L] - \frac{\gamma}{2}\text{var}(w|e_S, e_L) - \frac{\delta_S}{2}(e_S)^2 - \frac{\delta_L}{2}(e_L)^2$

That is, the board picks the contract to maximize the expectation of the terminal value minus the manager’s wage (i.e., the firm’s expected residual value) taking into consideration how the manager’s short-term and long-term efforts depend upon the parameters of the contract (via the incentive compatibility constraint IC), such that, at the optimal levels of effort, the manager achieves at least his reservation utility (as in the participation constraint PC).

F. Discussion

There are a couple of features of the above environment to note. First, since the terminal value of the firm is simply the sum of the realized values of both the short-term and long-term projects, both projects contribute equally to the long-term value of the firm. Thus, the long-term project is no better or more important than the short-term project. Second, short-term value does not lead to short-term results that the market may confuse with long-term value. Thus, in this model there is no manipulation, whereby manager takes actions to manipulate observable results in an attempt to fool shareholder into believing that long-term value (which is reflected in near-term stock price) is higher than it actually is. (That is, there is no signal jamming as in Narayanan (1985) and Stein (1989).) However, to the extent that the manager’s effort may, on average, be more productive if exerted on the long-term project (i.e., $\gamma_L > \gamma_S$), then a contract that induces the manager to exert more effort on the long-term project will produce more value. Thus, the issue is not whether the manager manipulates short-term performance, but whether the contracting environment is such that the equilibrium contract induces more managerial effort exerted on long-term projects when
\( \gamma_L > \gamma_S \).

Third, although the manager’s marginal productivity of effort on terminal value is \( \gamma_i \) (for \( i \in S, L \)), the marginal impact of the manager’s efforts on the manager’s total compensation depend upon the marginal impact of each type of effort on both the market price at \( t = 1 \) and the terminal value. The marginal impact of effort on the market price at \( t = 1 \) depends upon the equilibrium price function then, which in turn depends upon the aggressiveness with which informed traders trade on their private information about the manager’s effort with respect to a particular project. If more informed traders collect information on the short-term project, for example, then the competition among these traders will result in a price that reflects more of this type of information than regarding the long-term project. In addition, if there is more noise in investors’ signals of the long-term project’s value than for the short-term project (i.e., \( \sigma_{\varepsilon_L}^2 > \sigma_{\varepsilon_S}^2 \)), then investors trades will be less informative about the long-term project (holding constant the amount of intrinsic uncertainty \( \sigma_{\eta_S}^2 = \sigma_{\eta_L}^2 \)).

III. Equilibrium in the Securities Market (Taking the Contract as Given)

A. The Benchmark Case: First-Best Effort (No Agency Problem)

Before the equilibrium in the securities market is characterized, we first specify the first-best effort levels and value as a benchmark for which to compare the equilibrium levels of effort and value.

LEMMA 1: The first-best level of effort and the resulting expected terminal value (net of the cost of compensation for the disutility of effort) is as follows:

\[
\begin{align*}
\epsilon_{FB}^S &= \frac{\gamma_S}{\delta_S} \quad (12) \\
\epsilon_{FB}^L &= \frac{\gamma_L}{\delta_L} \quad (13)
\end{align*}
\]
and the resulting residual value is

\[ E(v) - E(w) = \frac{\gamma_S^2}{2\delta_S^2} + \frac{\gamma_L^2}{2\delta_L^2} \]  \hspace{1cm} (14)

Proof: See the appendix.

Lemma 1 is intuitive: the manager’s short-term and long-term efforts should be higher, the higher the marginal productivity of effort \( \gamma_i \), and the lower the marginal disutility of each type of effort \( \delta_i \) (for \( i \in S, L \)). The unconditional expectation of the expected residual value is simply \( E[E(v) - E(w)] = \frac{\gamma_S^2}{2\delta_S^2} + \frac{\gamma_L^2}{2\delta_L^2} \).

B. The Price Function

In order to determine the manager’s effort as a function of the contract terms offered, we first derive the secondary market price function. The following lemma specifies an expression of the price as a function of the contract terms – given that the manager’s compensation depends on that price (and the expected terminal value).

LEMMA 2: The price function at \( t = 1 \), given by equation (6) \( \text{[6]} \left( \text{ie., } p = p_0 + \lambda \Psi \right) \), is as follows:

\[
p_0 = \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left[ E[v] - \frac{\omega_0}{1 - \omega_v} \right],
\]

\[
\lambda = \left( \frac{1 + \omega_v}{1 + \omega_p} \right) \left( \frac{\text{cov}(v, \Psi)}{\text{var}(\Psi)} \right)
\]

where \( E[v] \) depends upon the manager’s equilibrium levels of effort (specified below).

Proof: See the appendix.

For a fixed expected gross terminal value \( E[v] \) and a fixed wage \( \omega_0 \), the price is decreasing in both \( \omega_v \) and \( \omega_p \). This is true since the higher either \( \omega_v \) or \( \omega_p \), the higher is the manager’s
pay (which reduces the residual value retained by the shareholders). However, the values of \( \omega_v \) and \( \omega_p \) determine the manager’s incentive to exert effort, which increases \( E[v] \). In addition, they also affect the amount of risk in the manager’s compensation, which affects the fixed wage \( \omega_0 \) that must be paid to induce the manager to take the job in the first place.

In order to determine the variance of the net order flow \( \Psi \) and the correlation between the net-order-flow and the gross terminal value \( v \), the optimal trades of the informed investors must be determined. We conjecture that the trades of investors who are informed about either the short- or long-term project will be linear in the unexpected component of their signal (i.e., either \( \theta_s - E(\theta_s) \) or \( \theta_L - E(\theta_L) \)). If the trades of informed investors are linear in their signals, then the net order flow \( \Psi \) will also be linear in these signals (and the trades of the liquidity traders). Thus, we assume that

\[
\Psi = \pi_S [\theta_S - E(\theta_S)] + \pi_L [\theta_L - E(\theta_L)] + Z,
\]

where \( \pi_S \) and \( \pi_L \) are, as yet, undetermined coefficients that quantify the sensitivities of the collective trades of the investors informed about the short-term and long-term project, respectively. Given this, the price function can be written as

\[
p = p_0 + \lambda \left( \pi_S [\theta_S - E(\theta_S)] + \pi_L [\theta_L - E(\theta_L)] + Z \right)
\]

where \( \lambda_S \equiv \lambda \pi_S \) and \( \lambda_L \equiv \lambda \pi_L \). Thus, the sensitivity of the price to a given signal \( \theta_i \) depends upon both the sensitivity of the collective informed investors’ trades to that signal \( \pi_i \) and the resulting correlation between all of the net order flow \( \Psi \) and the terminal value

\[
\lambda = \left( \frac{1+\omega_v}{1+\omega_p} \right) \left( \frac{\text{cov}(v, \Psi)}{\text{var}(\Psi)} \right).
\]

If \( \theta_i \) is completely uninformative, informed investors’ trades will not be sensitive to that signal, resulting in \( \pi_i = 0 \) and \( \lambda_i = 0 \) (even though \( \lambda \neq 0 \)). If, relative to the completely uninformative–signal case, the signal is more informative and the collective trades of the informed investors are more sensitive to that signal, then the price is more sensitive to that signal. Of course, the sensitivity of the collective trades of the informed investors depends upon the number of investors who obtain that signal and how
sensitive the optimal demand of a typical informed investor is to that signal.

C. Optimal Effort Conditional on the Contract and Prices

Before we can completely specify the price function, first we must determine the optimal effort the manager exerts on each of the two types of projects given his compensation so that the unconditional expected gross terminal value $E[v]$ can be specified. For this we take the form of the price function from Lemma 2 as given. Later, to the extent that the parameters of the price function depend upon the managerial effort levels and the contract offered by the shareholders, we will have to solve for reduced-form fixed point for the price function that is internally consistent. The following lemma, providing the first step in this process, specifies the optimal effort levels given arbitrary parameters for both the manager’s compensation contract and the price function.

LEMMA 3: For a given contract $\omega = (\omega_0, \omega_p, \omega_v)$ and the parameters $(\lambda_S, \lambda_L, \lambda)$ of the $t = 1$ price function as in equation (15), the optimal effort levels (for $i = S$ and $L$) are:

$$\hat{e}_i(\omega) = \frac{\gamma_i}{\delta_i} W_i.$$ (16)

where $W_i \equiv \omega_p \lambda_i + \omega_v$.

Proof: See the appendix.

Trivially, the higher is the marginal productivity of type-$i$ effort (i.e., $\gamma_i$), the greater the manager’s incentive to exert type-$i$ effort. This is true since any amount of type-$i$ effort affects both the terminal value $v$ and (via the signals of the value of projects) the intermediate price $p$; for positive contract weights placed on either value (i.e., $W_i > 0$), the more productive that effort (i.e., the greater $\gamma_i$), the greater is the manager’s incentive to exert that effort. Also, since $\delta_i$ is the manager’s marginal cost of effort spent on project $i$, the lower $\delta_i$, the greater the incentive to exert effort on project $i$. 

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The variable $W_i$ specifies the extent to which the manager’s effort on project $i$ affects his total compensation: (1) The manager’s effort on project $i$ increases the informed investors’ signal on project $i$, which (a) raises those informed investors’ trades by $\pi_i$ and (b) raises the near-term price by $\lambda_i \equiv \lambda \pi_i$; given the contract weight $\omega_p$ on the near-term price, the manager’s compensation thus rises by $\omega_p \lambda_i$. (2) The manager’s effort on project $i$ also increases the expected gross terminal value directly; given the contract weight $\omega_v$ on the gross terminal value, the manager’s compensation rises by $\omega_v$. Thus, the manager’s efforts on both the short-term and the long-term projects are increasing in the contract’s weights $\omega_p$ and $\omega_v$. But, for a fixed set of contract weights $(\omega_p, \omega_v)$, the manager will exert relatively more effort on the project that has a larger $\lambda_i$, which depends upon the number of investors obtaining the type-$i$ signal and the aggressiveness with which they collectively trade on that signal. The greater the relative sensitivity of the collective trades to the type-$i$ signal, the greater the effort on the type-$i$ project.

Weight placed on the terminal value is relatively more effective at inducing effort than weight placed on intermediate price (i.e., $\frac{\partial e_i}{\partial \omega_p} < \frac{\partial e_i}{\partial \omega_v}$) because the terminal value exactly reflects the manager’s effort whereas the intermediate price only reflects his effort multiplied by the regression coefficient $\lambda_i < 1$. Thus, since the terminal value is more sensitive to a given amount of type-$i$ effort than the intermediate price, the manager will exert more type-$i$ effort per-unit of weight placed on the terminal value than on the intermediate price.

Finally, if $\omega_p = 0$ and $\omega_v > 0$, then the equilibrium levels of effort will be proportional to the first-best levels (specified in Lemma 1); the agency problem uniformly reduces effort, but does not reduce the effort of one project relatively more than the other. However, if $\omega_p > 0$, then there will exist variation in the relative reduction of one type of project compared to the other due to differences in the extent to which the market reflects information on the two types of projects. The size of this distortion depends upon the variation in $\lambda_i$ and the amount of weight $\omega_p$ placed on the near-term price. Thus, incentive compensation based on near-term price is potentially distortive.
D. Optimal Speculative Trades

Notice that the information structure assumed above is such that signals provide information on both the managers effort (i.e., the component $\gamma_i e_i$ in the signal $\theta_i = \gamma_i e_i + \eta_i + \varepsilon_i$) and the component of the outcome that is beyond the manager’s control (i.e., $\eta_i$). Given the noise $\varepsilon_i$ in the signal, informed traders optimally extract the information content of the signal with respect to both $\gamma_i e_i$ and $\eta_i$, both of which have relevance with respect to future value. However, with respect to the manager’s information set, both $\varepsilon_i$ and $\eta_i$ represent risk to the manager. While the optimal extraction of the information content from the signal by informed traders minimizes (but cannot eliminate) the effect of the noise, the informed traders want to use the information in the signal on $\eta_i$. Thus, there is a benefit and a cost associated with the number of informed traders in a particular type of signal being high: the price reflects less of the noise (which makes contracts written on the near-term price more efficient and, thus, less costly) but it also reflects more information about $\eta_i$ (which makes contracts written on the near-term price riskier to the manager and, thus, more costly).

Given optimal levels of effort for the short- and long-term projects as a function of the price function parameters and the incentive contract, the expectation of the terminal residual value conditional on the net order flow can be calculated under the assumption that optimal informed trades are linear in the signals. Given these conditional expectations, it can then be verified that the optimal demands of informed traders are indeed linear in their signals. This then allows us to characterize the equilibrium price function and the resulting incentives they provide. Lemma 4 below specifies the optimal trades of informed traders given a price function of the form specified in equation (15); Lemma 5 (in the next sub-section) specifies the equilibrium price function given these optimal demands.

**Lemma 4:** The optimal trade $X_i^*$ of an individual trader who observes $\theta_i$ (for $i \in S, L$) is
as follows:

\[ X_i^* = \frac{W_i}{\delta_i} E[\gamma_i^2 - \bar{\gamma}_i^2 | \theta_i] + E[\eta_i | \theta_i] }{ \lambda (N_i^I + 1) \left( \frac{1 + \omega_p}{1 - \omega_v} \right) } \]  

where

\[ E[\gamma_i^2 - \bar{\gamma}_i^2 | \theta_i] = \mu_i (\theta_i - E[\theta_i]) = \mu_i \left( \frac{W_i}{\delta_i} (\gamma_i^2 - \bar{\gamma}_i^2) + \eta_i + \varepsilon_i \right) \]  

\[ E[\eta_i | \theta_i] = \mu_{\eta_i} (\theta_i - E[\theta_i]) = \mu_{\eta_i} \left( \frac{W_i}{\delta_i} (\gamma_i^2 - \bar{\gamma}_i^2) + \eta_i + \varepsilon_i \right) \]  

\[ \mu_i \equiv \frac{W_i}{\delta_i} \sigma_{\gamma_i}^2 \left( \frac{\sigma_{\gamma_i}^2 + \sigma_{\eta_i}^2 + \sigma_{\varepsilon_i}^2}{\sigma_{\gamma_i}^2} \right) \]  

\[ \mu_{\eta_i} \equiv \frac{W_i}{\delta_i} \sigma_{\eta_i}^2 \left( \frac{\sigma_{\gamma_i}^2 + \sigma_{\eta_i}^2 + \sigma_{\varepsilon_i}^2}{\sigma_{\eta_i}^2} \right) \]  

**Proof**: See the appendix.

Given these demands, the collective trades of the informed traders imply that the price varies with the signals according to

\[ \lambda (N_i^I X_i^* + N_i^L X_i^*) = \lambda_S (\theta_S - E[\theta_S]) + \lambda_L (\theta_L - E[\theta_L]), \]  

where \( \lambda_S = \left( \frac{N_i^S}{N_i^S + 1} \right) \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left( \frac{W_S}{\delta_S} \mu_S + \mu_{\eta_S} \right) < 1 \) and \( \lambda_L = \left( \frac{N_i^L}{N_i^L + 1} \right) \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left( \frac{W_L}{\delta_L} \mu_L + \mu_{\eta_L} \right) < 1 \).

Thus, the relative sensitivity of the price to each of the two types of signals (i.e., \( \lambda_S \) relative to \( \lambda_L \)) depends on (1) the number of each type of informed traders (i.e., the size of \( \frac{N_i^I}{N_i^S + 1} \) relative to \( \frac{N_i^L}{N_i^L + 1} \)) and (2) the size of \( \frac{W_S}{\delta_S} \mu_S + \mu_{\eta_S} \) relative to \( \frac{W_L}{\delta_L} \mu_L + \mu_{\eta_L} \). The following corollary specifies which type of signal is reflected most in the price.

**COROLLARY 1**: If the short- and long-term projects are the same (in terms of \( \delta_i, N_i^I, \sigma_{\gamma_i}^2, \bar{\gamma}_i^2 \), and \( \sigma_{\eta_i}^2 \)) except that the long-term signal is noisier than the short-term signal (i.e., \( \sigma_{\varepsilon_L}^2 > \sigma_{\varepsilon_S}^2 \)), then for a given set of contract weights \( (\omega_p, \omega_v) \), the price at \( t = 1 \) will reflect more of the signal on the short-term project than the signal on the long-term project.

**Proof**: See the discussion below.
The values of $\mu_i$ and $\mu_{\eta_i}$ quantify the sensitivity of the expectations of $\gamma_i^2$ and $\eta_i$, respectively, to the signal $\theta_i$. The higher the quality of the signal on project $i$, the less noise in the signal (i.e., $\sigma^2_{\varepsilon_i}$ is smaller) and the greater $\mu_i$ and $\mu_{\eta_i}$. Importantly, the sensitivity of informed expectation is also increasing in the contract weights $(\omega_p, \omega_v)$ through $W_i \equiv \omega_p \lambda_i + \omega_v$. Thus, informed expectations depend on the contract and, ceteris paribus, will be relatively more sensitive to the short- (long-) term signal if $\lambda_S > (\lambda_L)$. That is, a larger $\lambda_i$ reinforces itself. Thus, if $\sigma^2_{\varepsilon_L} > \sigma^2_{\varepsilon_S}$, we have $\mu_S > \mu_L$ and $\mu_{\eta_S} > \mu_{\eta_L}$. Then, $\lambda_S$ will be large (due to the direct effect), which will further increase $\lambda_S$, resulting in the price reflecting a large amount of the short-term project signal. When that happens, the manager will have a more intense incentive to exert effort on the short-term project (relative to the long-term project) since $\lambda_S > \lambda_L$.

Of course, since the profits to be gained and the costs to be incurred vary across the types of signals, $N^I_S$ need not equal $N^I_L$. For example, if, with $N^I_S = N^I_L$ and $\sigma^2_{\varepsilon_L} > \sigma^2_{\varepsilon_S}$, the price reflects more of the short-term signal, then the profits to be gained from collecting and trading on the short-term signal will be less than those associated with the signal on the long-term project. As a result, unless the cost of the signal on the short-term project is less than that for the long-term project, then either speculators collecting short-term information will have to exit or speculators collecting long-term information will enter, resulting in $N^I_S < N^I_L$. This then results in the price reflecting less short-term information and more long-term information which could improve the efficacy of the price at inducing effort on the long-term project.

Lemma 5 in the next section specifies the equilibrium price function for given numbers of short-term and long-term informed traders. Given that price function, the expected trade profits for each type of trader can be derived. As will be shown, the aggressiveness with which each type of informed trader trades on their information depends upon the contract weights. This implies that the contract weights will have an influence on the informativeness of the price with respect to each type of signal, which will affect the cost of inducing effort
via the near-term price versus the long-term value.

E. Equilibrium Price Function

Given the demands specified in Lemma 3, the price function at \( t = 1 \) is provided next.

**LEMMA 5:** The price function at \( t = 1 \) is

\[
p = p_0 + \lambda_S \gamma_S (e_S - \bar{e}_S) + \lambda_L \gamma_L (e_L - \bar{e}_L) + \dot{\eta}_S + \dot{\varepsilon}_S + \dot{\eta}_L + \dot{\varepsilon}_L + z \tag{22}
\]

where

\[
p_0 = \left( \frac{1 - w}{1 + \omega_p} \right) [\bar{v}_S + \bar{v}_L] - \frac{\omega_0}{1 + \omega_p}
\]

with, for \( i \in S, L \)

\[
\bar{e}_i = \frac{\bar{v}_i}{\delta_i} W_i, \quad \bar{v}_i = \frac{\bar{v}_i^2}{\delta_i} W_i, \quad \dot{\eta}_i = \lambda_i \eta_i, \quad \dot{\varepsilon}_i = \lambda_i \varepsilon_i, \quad z = \left( \frac{1 - w}{1 + \omega_p} \right) \Gamma Z
\]

\[
\Gamma = \frac{1}{\sigma_Z} \left[ \sum_{i \in S, L} \left( \left( \frac{W_i}{\lambda_i} \right)^2 \sigma^2_{\gamma_i} + \sigma^2_{\eta_i} \right) \left( \frac{N_i^M M_i}{N_i^M + 1} \right) \left( 1 - \frac{N_i^M M_i}{N_i^M + 1} \right) \right]^{1/2}
\]

\[
M_i \equiv \frac{W_i}{\delta_i} \mu_i + \mu_{\eta_i}
\]

and, \( \lambda_i \) is the fixed point value that satisfies the following expression (given that \( \mu_i \) and \( \mu_{\eta_i} \) from Lemma 4 depend upon \( \lambda_i \) via \( W_i \) defined in Lemma 3):

\[
\lambda_i = \frac{\left( \frac{N_i^M}{N_i^M + 1} \right) \left( \frac{\omega_0}{\delta_i} \mu_i + \mu_{\eta_i} \right)}{\left( \frac{1 + \omega_p}{1 - \omega_p} \right) - \left( \frac{N_i^M}{N_i^M + 1} \right) \left( \frac{\omega_0}{\delta_i} \mu_i \right)}
\]

**Proof:** See the appendix.

The equilibrium level of \( \lambda_i \) depends upon the number of informed speculators trading on information in project \( i \) according to the following corollary.
COROLLARY 2: The value of $\lambda_i$, the sensitivity of the price to speculators expectations of the manager’s effort in project $i$, increases with the number of speculators receiving information about project $i$.

Thus, if there are more speculators obtaining information on the short-term project, then the price will reflect more of their expectation of the manager’s short-term effort. As a result, if there are more speculators obtaining short-term information on the productivity of the short-term project, since the near-term price is more sensitive to that information, the near-term price induces the manager to exert (relatively) more effort on the short-term project. As is discussed in Section F below, the question is whether the manager’s greater focus on the short-term project induces speculators to also focus more on the short-term project (i.e., inducing more speculators to collect information on the short-term project). If so, then the increase in the number of speculators collecting short-term information reinforces the incentive of the manager to exert relatively more effort on the short-term project.

Before we examine how the expected profits of speculators depend upon the contract and the quality of information available, we first need to relate the initial price of the firm (i.e., the expected residual value) to the contract and the price function parameters. This will allow us to see the trade-off shareholders face when setting the contract weights, given the potential effects of the contract weights on the price process. Given the form of the price function (as in equation [22]), the expected residual value can be written as a function of the parameters of the price function and the incentive contract. Since the participation constraint will bind, $\omega_0$ will be a function of $\omega_p$ and $\omega_v$ (and the exogenous parameters), and the optimal contract will simply be a function of $\omega_p$ and $\omega_v$ (with $\omega_0$ set as a function of $\omega_p$ and $\omega_v$). Specifically, we have the following result:

LEMMA 6: For $i = S, L$, and for given values of $\omega_p$ and $\omega_v$, the fixed wage that satisfies the
participation constraint is

\[ \omega_0^* = (1 + \omega_p) \left\{ \frac{\gamma}{2} \left[ \sum_{i \in \{S, L\}} \left( W_i^2 \sigma^2_{\delta_i} + \left( \omega_p \lambda_i \right)^2 \sigma^2_{\xi_i} \right) + \omega_p^2 \sigma^2_\delta + \omega_v^2 \sigma^2_{\eta_T} \right] \right\} + \frac{1}{2} \sum_{i \in \{S, L\}} \left[ \bar{\gamma}_i^2 W_i^2 \right] - \omega_p \left( \frac{1 + \omega_p}{1 - \omega_p} \right) \sum_{i \in \{S, L\}} \bar{\gamma}_i^2 W_i - \omega_v \sum_{i \in \{S, L\}} \bar{\gamma}_i^2 W_i \right\} \]  

(23)

The expected residual value \( E[RV] \) is given by

\[ E[RV] = E[v] - E[w] \]

\[ = \frac{1}{2} \left\{ \sum_{i \in \{S, L\}} \bar{\gamma}_i^2 \left( \omega_p \lambda_i + \omega_v \right) \right\} - \frac{\gamma}{2} \left\{ \sum_{i \in \{S, L\}} \left[ \left( \omega_p \lambda_i + \omega_v \right)^2 \sigma^2_{\eta_i} + \left( \omega_p \lambda_i \right)^2 \sigma^2_{\xi_i} \right] + \omega_p^2 \sigma^2_\delta + \omega_v^2 \sigma^2_{\eta_T} \right\} \]

where \( \lambda_i \) characterizes the price function in the securities market (see Lemma 5).

The expected residual value consists of fractions of each component of the first-best expected residual value minus the risk premium that must be paid to the manager to compensate for the risk in the incentive compensation. Although the expected gross future value is increasing in both \( \omega_p \) and \( \omega_v \), the risk premium is also increasing in these contract parameters. A given increase in \( \omega_p \) increases the expected terminal value \( E[v] \) by \( \frac{\bar{\gamma}_S^2 \lambda_S}{\delta_S} + \frac{\bar{\gamma}_L^2 \lambda_L}{\delta_L} \). Thus, an increase in the sensitivity of the manager’s compensation to the near-term price increases the expected terminal value via both the short-term and long-term project. However, since the near-term price is affected by liquidity shocks, the impact is moderated by \( \lambda_S \) and \( \lambda_L \). As a result, a same-sized increase in \( \omega_v \) increases the expected terminal value by more (since \( \frac{\bar{\gamma}_S^2}{\delta_S} + \frac{\bar{\gamma}_L^2}{\delta_L} > \frac{\bar{\gamma}_S^2 \lambda_S}{\delta_S} + \frac{\bar{\gamma}_L^2 \lambda_L}{\delta_L} \) ).
As is standard in a Kyle-type model, for given numbers of informed investors of each type, the variance of the price is not a function of the variance of liquidity trades; that is
\[
\sigma^2_z = \left( \frac{1 - \omega_v}{1 + \omega_p} \right)^2 \Gamma \sigma^2_Z = \left( \frac{1 - \omega_v}{1 + \omega_p} \right)^2 \left[ \sum_{i \in \{S,L\}} \left\{ \left( \frac{W_i}{\delta_i} \right)^2 \sigma^2_{\gamma_i} + \sigma^2_{\eta_i} \right\} \left( \frac{N_i M_i}{N_i^I + 1} \right) \left( 1 - \frac{N_i^I M_i}{N_i^I + 1} \right) \right]
\]
and \( \lambda_i \) is not a function of \( \sigma^2_z \). This is true because the informed traders alter the aggressiveness with which they trade on their information in response to changes in the amount of noise created by liquidity traders. As a result, for fixed numbers of informed investors, the component of the manager’s compensation based on the near-term price is not risky due to the noise created by liquidity shocks.

However, two sources of variability in the near-term price create risk in the manager’s compensation. First, the price variance is a function of the noise in the informed traders’ signals \( \varepsilon_i \); greater noise in the signal generates less volatility in the price since traders trade less aggressively on lower quality (i.e., more noisy) signals. Second, the number of speculators choosing to get information on each type of project affects the information content of their collective trades, which affects the extent to which the price reflects the signals of the manager’s effort in each project. The greater the number of speculators collecting information on a particular project, the more intense is the competition and the more their collective trades reflect the signal (and its noise). On net, if there are more speculators informed about the short-term project than the long-term project, then the variance of expected short-term effort conditional on the price is smaller than the conditional variance of long-term effort, making it relatively cheaper for a contract written solely on the near-term price to induce short-term effort. Thus, if the market sustains a higher number of informed traders in a particular type of project, then that type of project will be “encouraged” in the sense that it is relatively more efficient, given the cost of inducing effort, to induce effort in that project. Again, a critical variable in determining if the short- or long-term project is excessively encouraged by the market is the equilibrium number of short- or long-term
informed speculators, which is taken up in the next section.

When $\sigma_{\eta T}^2 = 0$, the optimal contract will solely consist of weight placed on the terminal value. That is, for any value of $\lambda_i > 0$, the expected residual value is maximized at $\omega_p = 0$. When $\sigma_{\eta T}^2 > 0$, solely placing weight on the terminal value will be expensive since the manager is forced to bear risk not present in the near-term price. Consequently, it is optimal to place some weight on $p$. In that event, the composition of the investors in terms of the type of signals they collect affects the efficacy of the price.

Also note that for a given set of contract weights, the expected residual value is quadratic and concave in the values of $\lambda_S$ and $\lambda_L$. Thus, for a given set of contract weights, there exists a maximal expected residual value. Figure 1 depicts how the expected residual value of the firm depends upon the price parameters $\lambda_S$ and $\lambda_L$ for a given contract. Figure 1 shows the level sets for various values of the expected residual value given variation in $\lambda_S$ and $\lambda_L$ when the short-term signal is less noisy than the long-term signal. Note that, given Corollary 2, such variation in $\lambda_S$ and $\lambda_L$ may be due to variation in the number of speculators obtaining short- and long-term signals. The arrows in the figure show the direction of increasing value. Since both $\lambda_S$ and $\lambda_L$ are less than 1, the level sets are depicted only under this restriction. Also, when the short-term signal is less noisy than the long-term signal, the maximal expected residual value is at the dot, where $\lambda_S > \lambda_L$. In that case, Lemma 3 implies that the manager exerts more effort on the short-term project than on the long-term project.

F. Equilibrium in the Information Market and Expected Trade Profits

The problem considered here is different from the problem in which there is a short-term price (which depends upon the liquidity trading in the short-term market) that reflects information about the short-term project and a long-term price (which depends upon the liquidity trading in the long-term market) that reflects information about the long-term project. In that case, the difference in the expected profits of the short-term and long-
term informed traders will depend upon the difference of the variance of liquidity trading in
the short-term and long-term markets. Specifically, those markets with a higher amount of
liquidity trading variance will sustain a higher number of informed traders, resulting in that
market reflecting more information about the effort for that market’s project; then, that
project will be the cheapest to generate incentives for.

But, in our model, and in actual markets in which shares in multi-project firms are traded,
separate markets for each project do not exist. Rather, there is a single set of liquidity trades
for a single firm that is comprised of both short-term and long-term projects. Thus, this
single set of liquidity trades is responsible for generating profitable trade opportunities for
informed traders who collect information on either (or both) the short-term or long-term
project. The issue, which is taken up next, is to identify the features in the near-term price
process that creates differential incentives to collect the different types of information.

Whether or not an investor obtains a specific type of signal for a specific firm depends
upon the expected profit and the cost associated with obtaining that signal. The equilibrium
number of investors who obtain the short-term signal (denoted $N^{I_S}$) and the equilibrium
number of investors who obtain signals of the long-term project (denoted $N^{I_L}$) must be such
that

\[
E[\Pi^{I_S}_S|N^{I_S}_S] > C_S > E[\Pi^{I_S}_S|N^{I_S}_S + 1]
\] (25)

and

\[
E[\Pi^{I_L}_L|N^{I_L}_L] > C_L > E[\Pi^{I_L}_L|N^{I_L}_L + 1]
\] (26)

That is, the equilibrium number of speculators of each type must be such that the last
speculator of either type to enter can cover the cost of obtaining their signal (i.e., no exist)
but that any additional speculator that enters will make insufficient trade profits to cover
the cost of the signal (no entry).
Given the price function from Lemma 5, expected trade profits are given by the following lemma.

**LEMMA 7:** The expected profit of an informed trader who observes the signal \( \theta_i \) is

\[
E[\Pi_i | \theta_i] = E[X_m^I(v' - p)] = (1 - \omega_v) \left[ \frac{W_i}{\delta_i} \mu_i + \mu_{\eta_i} \right] \left( \frac{W_i}{\delta_i} \Delta_i + \mu_i + \eta_i + \varepsilon_i \right) \left[ \frac{1}{\Gamma(N_i^I + 1)^2} \right] \tag{27}
\]

where \( \Delta_i \equiv \gamma_i^2 - \gamma_i^2 \), for \( i = S \) or \( L \). Integrating over the possible values of the signal \( \theta_i \), the unconditional expected profit for a trader who will obtain a type-i signal (\( i = S \) or \( L \)) is

\[
E[\Pi_i] = \frac{(1 - \omega_v) M_i^2 \sigma_z}{(N_i^I + 1)^2} \left( \frac{(W_i/\delta_i)^2 \sigma_{\gamma_i}^2 + \sigma_{\eta_i}^2 + \sigma_{\varepsilon_i}^2}{\sum_{i \in \{S,L\}} \left[ \left( \frac{(W_i/\delta_i)^2 \sigma_{\gamma_i}^2 + \sigma_{\eta_i}^2}{\left( \frac{N_i M_i}{N_i^I + 1} \right) \left( 1 - \frac{N_i M_i}{N_i^I + 1} \right)} \right] \right]^{1/2} \right) \cdot M_i \sigma_z \tag{28}
\]

\[
E[\Pi_i] = \frac{(1 - \omega_v) M_i^2 \sigma_z}{(N_i^I + 1)^2} \frac{G_i}{\sqrt{2} \overline{H}_i} \left( \frac{H_i + \left( \frac{G_i}{\sqrt{2}} \right) H_{-i}}{2} \right), \text{ and } H_i = \frac{(N_i M_i)}{N_i^I + 1} \left( 1 - \frac{N_i M_i}{N_i^I + 1} \right)
\]

\[
G_i \equiv \left( \frac{W_i}{\delta_i} \right)^2 \sigma_{\gamma_i}^2 + \sigma_{\eta_i}^2, \quad \overline{H}_i \equiv \frac{H_i + \left( \frac{G_i}{\sqrt{2}} \right) H_{-i}}{2}, \text{ and } H_i = \left( \frac{N_i M_i}{N_i^I + 1} \right) \left( 1 - \frac{N_i M_i}{N_i^I + 1} \right)
\]

**Proof:** See the appendix.

As can be verified by equation (28), the expected profit of either type of informed trader is increasing in the variance of liquidity trades \( \sigma_z^2 \). If the variance of liquidity trades increases, then the number of informed traders in each type of signal will increase until the increased profits generated by the greater amount of liquidity trading is competed away and the expected profit from obtaining information on project \( i \) drops to the signal cost \( C_i \). Furthermore, as in the Kyle (1985) model, as the number of informed in a particular
signal increases, the market price reflects more of the information content of the signal these informed traders obtain. Thus, as $\sigma^2$ increases, the number of informed in both the short- and the long-term projects increases, making the price more informative with respect to both short- and long-term effort, which improves the efficacy of contracts written on the near-term price at inducing the manager to exert effort. This result is similar to that in Holmstrom and Tirole (1993), except that here we have two projects (and the price creates the incentive for the manager to exert effort on both projects rather than just one).

The following set of corollaries show how the expected profits depend upon the characteristics of the signals and the contract.

COROLLARY 3: If $N^I_S = N^I_L = N$ and the short- and long-term projects are the same (i.e., $\sigma^2_{\gamma_S} = \sigma^2_{\gamma_L} = \sigma^2_{\gamma}, \delta_S = \delta_L = \delta, \sigma^2_{\varepsilon_S} = \sigma^2_{\varepsilon_L} = \sigma^2_{\varepsilon}$) except that the $i$-th project has more uncertainty beyond the manager’s control (i.e., $\sigma^2_{\eta_i} > \sigma^2_{\eta_{-i}}$), then $\lambda_i > \lambda_{-i}, G_i > G_{-i}, M_i > M_{-i}, H_i > H_{-i}, \bar{H}_i < \bar{H}_{-i}, E[\Pi_i] > E[\Pi_{-i}], \frac{\partial E[\Pi_i]}{\partial \omega_v} < \frac{\partial E[\Pi_{-i}]}{\partial \omega_v}$ and $\frac{\partial E[\Pi_i]}{\partial \omega_p} < 0$, and $\frac{\partial E[\Pi_{-i}]}{\partial \omega_p} < 0$.

Since $E[\Pi_i] > E[\Pi_{-i}]$ when $N^I_S = N^I_L = N$, then it must be the case that $N^I_i > N^I_{-i}$ if the information market is in equilibrium. As a result, the price will reflect more project $i$ information, inducing the manager to exert relatively more effort on project $i$ than on $-i$. For example, if there is more uncertainty beyond the manager’s control in the long-term project, *ceteris paribus*, the information market will induce more effort in the long-term project. But, the corollary also indicates that if the Board increases either $\omega_v$ or $\omega_p$ (perhaps in an attempt to induce the manager to exert more effort on one or both projects), then there will be a relative drop in the expected profits associated with trading on the signal in the project with the greater uncertainty beyond the manager’s control. This will result in an exit of speculators obtaining that signal, resulting in the near-term price being a less precise indicator of the effort exerted on that project and, as a result, less effective at inducing effort on that project. Thus, an increase in $\omega_v$ or $\omega_p$ reduces the relative distortion in the incentive to exert effort on the project with the greater uncertainty beyond the manager’s control.
The above corollary assumes that the amount of noise in the short- and long-term project signals is the same. The following corollary considers when signal noise varies by project type.

COROLLARY 4: If $N_I^S = N_L^I = N$ and the short- and long-term projects are the same (i.e., $\sigma^2_{\gamma_S} = \sigma^2_{\gamma_L} = \sigma^2_{\gamma} = \delta$, $\sigma^2_{\eta_S} = \sigma^2_{\eta_L} = \sigma^2_{\eta}$) except that the $i$-th signal is noisier (i.e., $\sigma^2_{\epsilon_i} > \sigma^2_{\epsilon_{-i}}$), then $\lambda_{-i} < \lambda_i$, $G_{-i} < G_i$, $M_{-i} < M_i$, $H_{-i} < H_i$, $\bar{H}_{-i} > \bar{H}_i$, $E[\Pi_{-i}] < E[\Pi_i]$, $\frac{\partial E[\Pi_{-i}]}{\partial \omega_v} > \frac{\partial E[\Pi_i]}{\partial \omega_v}$.

For example, if there is more noise in the signal of the long-term project, then an increase in $\omega_v$ will make the expected profit associated with trading on the long-term signal increase. As a result, an increase in $\omega_v$ results in more long-term informed and fewer short-term informed. Thus, the statement that there will be less of a bias toward short-term projects if more weight is placed on far-term price is correct. But, increasing weight on the far-term price may result in the manager’s compensation being riskier (due to $\eta_T$). Thus, although there will be more long-run informed speculators making the intermediate price more informative with respect to the long-term project, that increased information content comes from placing relatively more weight on the far-term value rather than on the near-term price.

COROLLARY 5: The unconditional expected profit $E[\Pi_i]$ of speculators receiving a type-$i$ signal is decreasing in the number of speculators ($N_I^i$) receiving that signal and increasing in the number of speculators ($N_{-i}^I$) receiving the other type of signal.

Note that $\frac{\partial H_i}{\partial N_i^i} < 0$ for $N_i^I > 1$. Thus, $\frac{\partial H_i}{\partial N_{-i}^i} < 0$ and $\frac{\partial H_i}{\partial N_{-i}^I} < 0$. Then, since $E[\Pi_i]$ is decreasing in $\bar{H}_i$, we have $\frac{\partial E[\Pi_i]}{\partial N_{-i}^i} > 0$, $\frac{\partial E[\Pi_i]}{\partial N_{-i}^I} < 0$ since the effect of an increase in $N_i^I$ on the $(N_i^I + 1)^2$ term in the denominator of the expected profit is larger than the effect of an increase in $N_i^I$ on the drop in $\bar{H}_i$ in the denominator of the ratio under the radical in the expected profit expression.

COROLLARY 6: For any given contract and liquidity trade variance, there are multiple equilibria in the information market. In particular, there are multiple pairs of $N_L^I$ and $N_S^I$.
that satisfies the conditions for equilibrium in the information market. The set of equilibrium $N_I^L$ and $N_I^S$ pairs is such that there is an inverse relationship between the equilibrium number of speculators obtaining one type of signal and the number obtaining the other type of signal. That is, there is a crowding out in the sense that if in one equilibrium there are more speculators getting one type of signal than in another equilibrium, then the number of speculators getting the other type of signal is less than in the other equilibrium. So, if the market is currently in an equilibrium in which there are many speculators obtaining the signals on the short-term project, then there will be relatively few speculators collecting information on the long-term project.

Consider a combination $(N_I^S, N_I^L)$ such that $E[\Pi_S|N_I^S, N_I^L, (\omega_p, \omega_v)] > C_S > E[\Pi_S|N_I^S + 1, N_I^L, (\omega_p, \omega_v)]$ and $E[\Pi_L|N_I^S, N_I^L, (\omega_p, \omega_v)] > C_L > E[\Pi_L|N_I^S, N_I^L + 1, (\omega_p, \omega_v)]$. Holding fixed $N_I^L$, if the value of $N_I^S$ is increased (to $N_I^S' > N_I^S$), then $E[\Pi_S|N_I^S', N_I^L, (\omega_p, \omega_v)]$ drops. If this drop is large enough such that $E[\Pi_S|N_I^S', N_I^L, (\omega_p, \omega_v)] < C_S$, then the equilibrium with $N_I^L$ no longer exits. However, by the lemma above, $E[\Pi_S|N_I^S, N_I^L, (\omega_p, \omega_v)]$ is decreasing in $N_I^L$; thus, for $N_I^S'$, it must be the case that the number of long-term informed speculators must drop in order to raise $E[\Pi_S|N_I^S', N_I^L, (\omega_p, \omega_v)]$ to be above $C_S$. Thus, there are multiple equilibrium combinations of $N_I^S$ and $N_I^L$, with an inverse relationship between the numbers of speculator informed about each type of project. This, combined with Corollary 2, implies that there is an inverse relationship between $\lambda_S$ and $\lambda_L$.

Let the set of possible combinations of $N_I^S$ and $N_I^L$ such that equations (25) and (26) hold for a given contract pair $(\omega_p, \omega_v)$ be denoted $\mathcal{N}(\omega_p, \omega_v)$. Let the $i$th particular combination be denoted $N^i(\omega_p, \omega_v) \in \mathcal{N}(\omega_p, \omega_v)$. Associated with this particular combination is an expected residual value (or initial price for the firm) $ERV(N^i(\omega_p, \omega_v))$. Also let $\Pi^{max}(\omega_p, \omega_v) = \max_i \left(ERV(N^i(\omega_p, \omega_v)) \right)$. Also let $N^{max}(\omega_p, \omega_v)$ denote the pair of values of $N^i(\omega_p, \omega_v)$ associated with the maximal expected residual value. For any choice of contract $(\omega_p, \omega_v)$, there are multiple equilibrium, only one of which maximizes the expected residual value of the firm. To see this, consider Figure 2. In that figure, the downward sloping dashed
lines indicate combinations of $\lambda_S$ and $\lambda_L$ (associated with the inversely related values of $N^I_S$ and $N^I_L$) that are associated with a particular contract. For each particular contract, each point on the line indicating the combinations of $\lambda_S$ and $\lambda_L$ resulting from equilibrium in the information market is associated with a different level set. The point where the highest level set is tangent to the $\lambda_S$ and $\lambda_L$ combination line is where the expected residual value is maximized; all other points on the $\lambda_S$ and $\lambda_L$ combination line are associated with lower expected residual values.

Corollary 3 and Corollary 6 together imply that if the short- and long-term projects are the same except that $\sigma^2_{\eta_L} \geq \sigma^2_{\eta_S}$ and $C_S \leq C_L$, then, for all of the combinations of $N^I_S$ and $N^I_L$ consistent with equilibrium in the information market, it is always true that $N^I_S \geq N^I_L$. Thus, if there is more uncertainty beyond the manager’s control in the long-term project, then the market will “encourage” effort in the short-term project because the market will reflect more information on the short-term project. Similarly, Corollary 4 and Corollary 6 imply that if the only difference between the short- and long-term projects is $\sigma^2_{\varepsilon_L} > \sigma^2_{\varepsilon_S}$ and $C_S \leq C_L$, then the information market equilibrium is such that $N^I_S \geq N^I_L$. Again, in this case, the market encourages the short-term project over the long-term project. It must be stressed, however, that in both of these cases, the market encourages the short-term project over the long-term project independent of the expected productivity of the manager’s effort in each of these projects. This, however, does not necessarily imply that the outcomes are inefficient; if the short-term signals are less noisy, then, given the cost of inducing effort, it may make sense to encourage short-term effort. The question is whether the market encourages short-term effort too much. The next section considers the optimality of equilibrium.

IV. The Equilibrium Incentive Contract

Lemma 7 and Corollary 6 above imply that if the number of informed speculators can adjust quickly, then the firm cannot, by picking the compensation contract alone, maximize the expected residual value of the firm. If the firm picks a particular contract, there will be
multiple pairs \((N^S, N^L)\) such that the expected trade profits just cover the information collection costs. For example, consider Figure 3 below. For a given set of primitive parameters (e.g., \(\gamma, \sigma^2, \epsilon_i\), etc) denoted \(\Omega_l \in \Omega\) (where \(\Omega\) is the set of all possible parameters), the line labeled \(E(\Pi|\omega_p, \omega_v, \Omega_l) = 0\) consists of \((N^S, N^L)\) pairs such that equations (25) and (26) hold when the contract is \((\omega_p, \omega_v)\). (Although these pairs are depicted by a continuous line, since the number of speculators is discrete, the actual relationship is a set of discrete points on that line.)

All along (the set of discrete points on) \(E(\Pi|\omega_p, \omega_v, \Omega_l) = 0\), the expected residual value of the firm varies, along with market liquidity and, as a result, the information content of the market price with respect to effort exerted on the two types of projects. Thus, if the number of informed speculators is not fixed, then any choice of contract \((\omega_p, \omega_v)\) merely determines a set of possible expected residual values for the firm (depending upon the specific \((N^S, N^L)\) pair that obtains). If there is a change in the underlying environment (say from \(\Omega = \Omega_0\) to \(\Omega = \Omega_1\)), there will be a shift in the set of \((N^S, N^L)\) pairs that satisfy equations (25) and (26) from \(E(\Pi|\omega_p, \omega_v, \Omega_0) = 0\) to \(E(\Pi|\omega_p, \omega_v, \Omega_1) = 0\). (In the case depicted, the change in the underlying environment causes expected profits to increase (for any given value of \((N^S, N^L)\)).

Given the change in the underlying environment, the change in the endogenous variables that occurs depends upon which variables can adjust more quickly. For example, consider what happens if, in the short run, (1) the contract is fixed (that is, the contracts are legal documents that take time to renegotiate) and (2) the total number of speculators who are trained to interpret and trade on information is fixed. While the total number of speculators is fixed at \(N^\text{Tot}_0 = N^S_0 + N^L_0\), it is likely that some long-term speculators may be able to quickly become short-term speculators (and vice versa). The line labeled \(N^\text{Tot}_0\) denotes all of the combinations of \(N^S\) and \(N^L\) such that \(N^\text{Tot}_0 = N^S + N^L\). Then, given the change in underlying environment from \(\Omega_0\) to \(\Omega_1\), the short-run outcome will be where \(E(\Pi|\omega_p, \omega_v, \Omega_1) = 0\) crosses \(N^\text{Tot}_0\). In Figure 3, since the \(E(\Pi|\omega_p, \omega_v, \Omega_l)\) lines are steeper than the \(N^\text{Tot}_0\) line,
the change in the environment that increases profits results in at least a short-run increase in the number of long-term speculators and a drop in the number of short-term speculators. But, note that, as in Figure 4, if the $E(\Pi|\omega_p, \omega_v, \Omega_l)$ lines are flatter than the $N_{0\text{Tot}}$ line, then such an environmental change creates a short-run increase in short-term speculators at the cost of a drop in long-term speculators.

Given the environmental change, and the resulting movement along $N_{0\text{Tot}}$, what happens to the incentive to change other variables that may be adjusted in the long run? For example, at the new point on $N_{0\text{Tot}}$, the initial contract (denoted $(\omega_p0, \omega_v0)$) may be sub-optimal; there may be an incremental gain in terms of expected residual value to renegotiating the contract to say $(\omega_p1, \omega_v1)$. Such a change will produce another shift in the $E(\Pi|\omega_p, \omega_v, \Omega_l)$ line (since the expected profits from each type of signal depend upon the parameters of the compensation contract). In fact, the new line may not even cross $N_{0\text{Tot}}$ at $(N_S0, N_L0)$. Thus, if, in the short-run, there is a fixity in $N_{0\text{Tot}}$ but flexibility in the mixture $(N^S, N^L)$, then the shareholders can pick the contract $(\omega_p, \omega_v)$ that maximizes expected residual value as long as $(N^S, N^L)$ is on $N_{0\text{Tot}}$. That is, for every pair $(N^S, N^L)$ on $N_{0\text{Tot}}$, the shareholders can find the contract $(\omega_p, \omega_v)$ that produces a $E(\Pi|\omega_p, \omega_v, \Omega_l)$ line that crosses $N_{0\text{Tot}}$ at that $(N^S, N^L)$ point. Associated with that contract will be an expected residual value. The shareholders should find the set of contracts for each $(N^S, N^L)$ pair on $N_{0\text{Tot}}$ and pick the single contract associated with the specific point on $N_{0\text{Tot}}$ that has the highest expected residual firm value.

Figure 5 illustrates the determination of the equilibrium contract. Let the current environment be described by $\Omega_1$. Consider a particular point on $N_{0\text{Tot}} : (N^S_0, N^L_0)$. There are multiple contracts that produce $E(\Pi|\omega_p, \omega_v, \Omega_1) = 0$ lines that go through the point $(N^S_0, N^L_0)$. Let this set of contracts be denoted $\omega(N^S_0, N^L_0)$ and let a particular member of this set be denoted $\omega_k(N^S_0, N^L_0) \in \omega(N^S_0, N^L_0)$. Each contract $\omega_k(N^S_0, N^L_0) \in \omega(N^S_0, N^L_0)$ produces a different expected residual value. Let the contract $(\omega_p0, \omega_v0) \in \omega(N^S_0, N^L_0)$ be the contract that is associated with the highest expected residual value among the expected residual values that obtain given this set of contracts. In the figure, the maximal
expected residual value for \((N^S_0, N^L_0)\) is labeled \(E(RV)_0\). Similarly, there exists a contract \((\omega_{p1}, \omega_{v1}) \in \omega(N^S_1, N^L_1)\) that produces a maximal expected residual value (subject to \((N^S_1, N^L_1)\)), labeled in the figure as \(E(RV)_1\). \(E(RV)_2\) is similarly derived. Let, for example, \(E(RV)_0 > E(RV)_1 > E(RV)_2\). Also let \(E(RV)_0\) be the largest of any expected residual value obtainable for all \((N^S, N^L) \in N^{Tot}_0\). If the shareholders know that there is a fixity in \(N^{Tot}_0\) but not in its composition, then the shareholders will be able to achieve \(E(RV)_0\) by picking the contract \((\omega_{p0}, \omega_{v0})\). That is, the shareholders pick the contract that maximizes expected residual value subject to producing \((N^S_0, N^L_0) \in N^{Tot}_0\). (Note this is true only if this contract is unique – that is, if this contract is not also the optimal contract for another combination \((N^S, N^L) \in N^{Tot}_0\). If this is not true, then there is an indeterminacy.)

On the other hand, consider what happens if there is less fixity in the total number of speculators than in the contract. Once again consider the change in environment to \(\Omega_1\). For concreteness consider the example in Figure 6, in which the change in environment immediately causes a drop in long-term speculators and an increase in short-term speculators. Before the shareholders can change the contract (which, by assumption, takes time), the expected profits under the old contract are positive and will create the incentive for more speculators or either type to enter. In this case, any of the combinations of \((N^S, N^L)\) on \(E(\Pi|\omega_{p0}, \omega_{v0}), \Omega_1) = 0\) will be feasible in the short-run. However, since there may be start-up or adjustment costs to entering, it is likely that the \((N^S, N^L)\) pairs that obtain will not be extreme relative to the initial pair \((N^S_0, N^L_0)\). For example, as indicated in Figure 6, let the new pair be \((N^S_1, N^L_1)\). Under this pair, however, \((\omega_{p0}, \omega_{v0})\) is likely not the optimal contract. Thus, given \((N^S_1, N^L_1)\), the shareholders will adjust the contract to be optimal, say to \((\omega_{p1}, \omega_{v1})\). But notice that there were many combinations of \((N^S, N^L)\) that are possible, with other combinations generating different optimal contracts and potentially lower expected residual values. That is, the shareholders are at the mercy of the entry decision of speculators, which are not determined by the contract choice of the shareholders. Thus, the shareholders essentially act as if they must take the number of speculators are
given. And if the shareholders have some influence, they have limited control. Thus, the concerns expressed by the Aspen Institute are understandable in the context of this model.

V. Corporate and Public Policy Remedies

In this section we examine the efficacy or justifications for a variety of corporate and/or public policies that can produce a Pareto improvement in the equilibrium. To keep the discussion that follows concrete, we focus on the situation in which there are only two differences between the short- and the long-term projects: (1) the noise in the long-term signal is greater than the noise in the short-term signal and (2) the expected productivity of the manager’s effort exerted on the long-term project is greater than that for the short-term project. In this situation, the market encourages the manager to exert more effort on the short-term project even though effort exerted on the long-term project is more productive.

One set of potential remedies is to subsidize the collection of information on the long-term project. Under the conditions specified above, the set of \((N_S, N_L)\) pairs consistent with equilibrium in the information market consist only of pair for which \(N_S > N_L\) since \(C_S = C_L\).

In this case, *aeternis paribus*, it may be possible to affect the numbers of speculators of each type to increase expected residual value. There are a couple of possible ways to achieve this possibility. First, the firm could subsidize information collection on the long-term project. It could do this by providing greater access to management concerned with the long-term project relative to managers concerned with the short-term project. If this greater access merely serves to lower the cost of collecting information, this policy is equivalent (from the information collection side) to subsidizing information collection in the long-term project. That is, let \(S_i\) denote the subsidy for the type-\(i\) project, \(i \in \{S, L\}\). Then, by setting \(S_L > 0\) and \(S_S = 0\), the effective cost of collecting information becomes \(C_S > C_L - S_L\). In the case of greater access, the cost to the firm associated with lowering the cost of information collection for the long-term project by \(S_L\) may cost less than \(S_L\); in the direct subsidy case, the firm’s cost will exactly equal \(S_L\). Thus, the firm will opt for a policy (greater access or...
direct subsidy) with the lowest cost.

The impact of the above (implicit/indirect or explicit/direct) subsidy is depicted in Figure 7 below. Without the subsidy, the set of equilibria is at the downward sloping dotted line labeled $E(\Pi|\omega_p, \omega_v), \Omega_0, T_S = T_L = S_S = S_L = 0) = 0$ going through point A. With the subsidy, the zero-expected profit combination line shifts right to the line labeled $E(\Pi|\omega_p, \omega_v), \Omega_0, T_S > 0, T_L = S_S = S_L > 0) = 0$, going through point B. If the equilibrium was a point A prior to the offer of the subsidy, then the equilibrium might move to point B. Whether the firm wants to do this or not depends upon whether the increase in expected residual value from A to B is larger than the total cost of the subsidy $N_1L^ST_L$. In the case shown, such a movement increases expected residual value (the dotted arrows indicate the direction of increasing level curves). Note that from a public policy perspective, if there is any increase in the expected residual value (even if $\Delta ERV < N_1L^ST_L$), the subsidy satisfies the Pareto Criterion since it is merely a transfer (from shareholders to speculators). Thus, under certain sets of parameters, there is a role for a centralized government.

As an alternative to a subsidy, a tax on short-term speculation will also allow for equilibria with $N^L > N^S$. Let $T_i$ denote the per-signal tax associated with signals in project $i$. If $T_S > 0$ and $T_L = 0$, then the expected zero profit combination line shifts down (as depicted in Figure 7 to the dotted line that goes through point C labeled $E(\Pi|\omega_p, \omega_v), \Omega_0, T_S > 0, T_L = S_S = S_L = 0) = 0$). In this case, the number of short-term speculators fall (from A to C), which reduces the amount of effort exerted on the short-term project induced by the near-term price. Although, without the tax, the amount the near-term price reflects the short-term project is too large relative to the long-term project, a drop in the extent to which the near-term price reflects the manager’s effort on the short-term project does reduce the efficacy of the near-term price at inducing short-term effort. Thus, although such a tax may reduce the relative distortion on short-term versus long-term effort, it nonetheless reduces total effort. As a result, the tax is justified only if the drop in value due to the reduction in total effort is more than offset by a gain created by reducing the relative bias created by the market.
plus the value of any tax revenue (redistributed in a manner independent of the collection of information). In the case depicted in Figure 7 this movement generates an increase in expected residual value. However, Figure 8 shows how a tax on short-term speculation may cause a drop in expected residual value.

Depending upon the circumstances, it may be easier to implement a tax versus a subsidy. For example, in order for the firm to implement an effective subsidy, it will need to tie the subsidy to the collection of long-term signals. Yet, it may be hard to differentiate between types of information. Alternatively, if the gains from trading on information concerning short-term projects will be realized over shorter intervals of time relative to those over which profits form long-term signals are made, then a tax on short-term capital gains and/or a forgiveness of tax on long-term capital gains will achieve the downward shift in the zero expected profit combination line as depicted in Figure 7.

One question is what would be the effect of a policy in which the firm publically discloses more information about one type of project relative to the other. An issue is whether that would potentially lead to an incentive to manipulate such statements (which might make the disclosures – and the resulting near-term price – less informative with respect to a particular type of project). Also, in order for the information to be credible, it must be collected and held privately so that the collection costs can be recouped via trading. If more information is publicly disclosed, then there may actually be less private information reflected in price.

VI. Conclusion

The paper develops a model in which the efficacy of incentive contracts and the information content of prices are jointly determined. Given the interaction between these variables, there are, for every possible contract, multiple combinations of numbers of short- and long-term speculators, with the expected residual value of the firm varying across these combinations. The paper shows that in some equilibria the market excessively encourages managers to exert effort on inferior short-term projects. The effectiveness of regulation designed to moderate
this encouragement will depend upon adjustment costs outside the model.
References


A. Appendix

Proof of Lemma 1: First-Best Results
When effort is contractible, the problem is as follows:

\[
\max E[v] - E[w]
\]

subject to \(E[u] = \bar{u} \), where \(\bar{u} \) is the agent's reservation utility (which, without a loss of generality, we set to \(\bar{u} = 0 \)). In the first best, the principal pays the agent his disutility level and the principal bears all the risk (the \(\text{var}(w) = 0 \)). This implies that the wage is simply

\[
E(w) = \frac{\delta S}{2} e_S^2 + \frac{\delta L}{2} e_L^2
\]

The principal’s problem reduces to

\[
\max \gamma_S e_S + \gamma_L e_L - \frac{\delta S}{2} e_S^2 - \frac{\delta L}{2} e_L^2
\]

The first order conditions with respect to \(e_S \) and \(e_L \) imply

\[
e_S^* = \frac{\gamma_S}{\delta S}
\]

\[
e_L^* = \frac{\gamma_L}{\delta L}
\]

The resulting residual value is thus

\[
E[v] - E[w] = \frac{\gamma^2_S}{2\delta S} + \frac{\gamma^2_L}{2\delta L}
\]

QED.

Proof of Lemma 2: The Price function at \(t = 1\)
The price is simply the conditional expectation of the residual value, which is the terminal value minus the wage payments made to the manager. The realized residual value is simply

\[
v' = v - w = v - [\omega_0 + \omega_p p + \omega_v] = (1 - \omega_v)v - \omega_0 - \omega_p p
\]

(A.1)

The unconditional expectation is thus

\[
E[v'] = (1 - \omega_v)E[v] - \omega_0 - \omega_p p_0
\]

where \(p_0 \equiv E[p] \). The price at \(t = 1\) is simply

\[
p = p_0 + \lambda \Psi
\]

(A.2)

where \(\lambda \Psi = E[v' - E[v']|\Psi] \). Since \(p_0 \equiv E[p] = E[v'] = (1 - \omega_v)E[v] - \omega_0 - \omega_p p_0 \) we have

\[
p_0 = \frac{1}{1 + \omega_p} \left( (1 - \omega_v)E[v] - \omega_0 \right)
\]

(A.3)

Note that the difference between the realized terminal residual value and its unconditional expectations (i.e., \(v' - E[v'] \)) is

\[
v' - E[v'] = v' - p_0 = (1 - \omega_v)v - \omega_0 - \omega_p p_0 - \omega_p \lambda \Psi - p_0
\]

\[
= (1 - \omega_v)v - \omega_0 - (1 + \omega_p) \left[ \frac{1}{1 + \omega_p} \left( (1 - \omega_v)E[v] - \omega_0 \right) - \omega_p \lambda \Psi \right]
\]

\[
= (1 - \omega_v)[v - E[v]] - \omega_p \lambda \Psi
\]

Thus

\[
\lambda \Psi = E[v' - E[v']|\Psi]
\]

\[
= E \left[ (1 - \omega_v) [v - E[v]] - \omega_p \lambda \Psi | \Psi \right]
\]

\[
= E \left[ (1 - \omega_v) [v - E[v]] \right] | \Psi - \omega_p \lambda \Psi
\]

\[
= \left( \frac{1 - \omega_v}{1 - \omega_p} \right) E \left[ v - E[v] | \Psi \right]
\]

(A.4)
Since $E[E[v]|\Psi] = E[v]$, the price function is simply

$$
p = \frac{1}{1 + \omega_p} \left[ (1 - \omega_v) E[v] - \omega_0 \right] + \left( \frac{1 - \omega_v}{1 + \omega_p} \right) E[v - E[v]|\Psi]
$$

$$
= \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left[ E[v] - \omega_0 \right] + \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \Gamma \Psi
$$

where $\Gamma \Psi = E[v - E[v]|\Psi] = \frac{\text{Cov}(v, \Psi)}{\text{var}(\Psi)} \Psi - E[\Psi]$. (i.e. $\Gamma = \frac{\text{Cov}(v, \Psi)}{\text{var}(\Psi)}$). Thus $p_0 = \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left[ E[v] - \frac{\omega_0}{1 - \omega_v} \right]$, and $\lambda \Psi \equiv \Gamma \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \Psi$. (i.e. $\lambda \equiv \Gamma \left( \frac{1 - \omega_v}{1 + \omega_p} \right)$) QED.

**Proof of Lemma 3: Optimal Short- and Long-term Effort as a Function of the Contract**

We conjecture that the trades of the informed agents are linear in their signals. Specifically, we conjecture that $\Psi$ is linear in the unexpected components of the signals (e.g., $\theta_S - E[\theta_S] = \gamma_S(e_S - e^*_S) + \eta_S + \epsilon_S$):

$$
\Psi = \pi_S \left( \theta_S - E[\theta_S] \right) + \pi_L \left( \theta_L - E[\theta_L] \right) + Z
$$

Since the price is linear in the net order flow, the price is also linear in the signals. Specifically,

$$
p = \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left[ E[v] - \frac{\omega_0}{1 - \omega_v} + \Gamma \Psi \right]
$$

$$
= \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left[ E[v] - \frac{\omega_0}{1 - \omega_v} + \Gamma \left( \pi_S (\theta_S - E[\theta_S]) + \pi_L (\theta_L - E[\theta_L]) + Z \right) \right]
$$

$$
= \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left[ \sum_{i=1}^{S+L} \left( \gamma_i e^*_i \right) \right] - \frac{\omega_0}{1 - \omega_v}
$$

$$
\pi \sum_{i=1}^{S+L} \left[ \pi_i (\gamma_i e - e^*_i) + \eta_i + \epsilon_i \right] + Z
$$

Conditional on the managers information, the price is distributed with the following mean and variance:

$$
E[p|e_S, e_L] = \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left( \sum_{i=1}^{S+L} \left( \gamma_i e^*_i \right) \right) + \sum_{i=1}^{S+L} \Gamma \pi_i \gamma_i (e_i - e^*_i) - \frac{\omega_0}{1 - \omega_v}
$$

$$
\text{Var}[p|e_S, e_L] = \left( \frac{1 - \omega_v}{1 + \omega_p} \right)^2 \left( \pi_S \Gamma \left( \sigma^2_{\theta_S} + \sigma^2_{\epsilon_S} \right) + \pi_L \Gamma \left( \sigma^2_{\theta_L} + \sigma^2_{\epsilon_L} + \sigma^2_{\eta_L} \right) \right)
$$

The incentive compatibility constraint is as follows:

$$
E[u] = E[w] - \frac{\gamma}{2} \text{var}(w) - \sum_{i=1}^{S+L} \frac{\delta_i}{2} e^2_i
$$

where

$$
E[w] = \omega_0 + \omega_p E[p|e_S, e_L] + \omega_v E[v]
$$

$$
= \omega_0 + \omega_p \left\{ \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left( \sum_{i=1}^{S+L} \left( \gamma_i e^*_i \right) \right) + \sum_{i=1}^{S+L} \Gamma \pi_i \gamma_i (e_i - e^*_i) \right\} - \frac{\omega_0}{1 - \omega_v}
$$

$$
+ \omega_v \sum_{i=1}^{S+L} \gamma_i e_i + \omega_v \sum_{i=1}^{S+L} \gamma_i e_i
$$

and

$$
\text{Var}[u|e_S, e_L] = \omega_p^2 \text{var}(p|e_S, e_L) + \omega_v^2 \text{var}(v|e_S, e_L) + 2\omega_p \omega_v \text{cov}(v, p|e_S, e_L)
$$

$$
= \omega_p^2 \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left( \pi_S \Gamma \left( \sigma^2_{\theta_S} + \sigma^2_{\epsilon_S} \right) + \pi_L \Gamma \left( \sigma^2_{\theta_L} + \sigma^2_{\epsilon_L} + \sigma^2_{\eta_L} \right) \right)
$$

$$
+ \omega^2_v \sum_{i=1}^{S+L} \sigma^2_{\gamma_i} + \omega_v \omega_p \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left( \pi_S \Gamma \sigma^2_{\theta_S} + \pi_L \Gamma \sigma^2_{\eta_L} \right)
$$

$$
= (W_S)^2 \sigma^2_{\theta_S} + (\omega_p \lambda_S)^2 \sigma^2_{\epsilon_S} + (W_L)^2 \sigma^2_{\theta_L} + (\omega_v \lambda_L)^2 \sigma^2_{\epsilon_L} + \sigma^2_{\eta_L} + (\omega_v)^2 \sigma^2_{\eta_T}
$$

Thus
Thus, the informed traders’ problem is equivalent to

Given the optimal effort functions, the problem is

The objective function can be rewritten as

The optimization problem faced by a trader who has a signal of the short-term project is

Proof of Lemma 4: The Optimal Trades of an Individual Trader

The objective function can be rewritten as

The FOC, \[ \frac{\partial E[u]}{\partial e_i} = 0, \] is

where \( \lambda_i \equiv \lambda \pi_i \). Thus, the optimal effort level (for \( i = S, L \)) are:

where \( W_i \equiv \omega_p \lambda_i + \omega_v = \omega_p \left( \frac{1 + \omega_v}{1 - \omega_v} \right) \pi_i \Gamma + \omega_v \). QED.

Proof of Lemma 4: The Optimal Trades of an Individual Trader

The optimization problem faced by a trader who has a signal of the short-term project is

The objective function can be rewritten as

Thus, the informed traders’ problem is equivalent to

Given the optimal effort functions, the problem is

The objective function can be rewritten as
\[
E[\Pi_S|\theta_S] = E\left[X_m^S\left(\left[\frac{\gamma_S^2}{\delta_S} W_S + \eta_S + \frac{\gamma_L^2}{\delta_L} W_L + \eta_L + \eta_T\right] - (E[\epsilon] + \Gamma \Psi)\right)|\theta_S\right]
\]
\[
= X_m^S\left[\frac{W_S}{\delta_S} E[\gamma_S^2 - \gamma_S^2 |\theta_S] + E[\eta_S|\theta_S] + \frac{W_L}{\delta_L} E[\gamma_L^2 - \gamma_S^2 |\theta_S] + E[\eta_L|\theta_S] + E[\eta_T|\theta_S]\right) - \Gamma E[\Psi|\theta_S] \]
\]
\[
= X_m^S\left[\frac{W_S}{\delta_S} E[\gamma_S^2 - \gamma_S^2 |\theta_S] + E[\eta_S|\theta_S]\right) - \Gamma \left( X_m^S + (N_S^L - 1)E[X_m^S|\theta_S]\right)]
\]

where \(X_m^S\) denotes the demands of the other investors who have the signal for the short-term project. The first order condition is

\[
\frac{\partial E[\Pi_S|\theta_S]}{\partial X_m^S} = \left[\frac{W_S}{\delta_S} E[\gamma_S^2 - \gamma_S^2 |\theta_S] + E[\eta_S|\theta_S]\right) - \Gamma \left( X_m^S + (N_S^L - 1)E[X_m^S|\theta_S]\right)] - X_m^S \Gamma = 0
\]

which implies

\[
X_m^S = \frac{1}{2\Gamma} \left[\frac{W_S}{\delta_S} E[\gamma_S^2 - \gamma_S^2 |\theta_S] + E[\eta_S|\theta_S]\right) - \Gamma (N_S^L - 1)E[X_m^S|\theta_S]\]

Since, in equilibrium, all investors that have the signal of the short-term project will have the same demand, the symmetric equilibrium demands are the value \(X_S^*\) such that \(X_m^S = E[X_m^S|\theta_S] = X_S^*\). This value is

\[
X_S^* = \frac{W_S}{\delta_S} E[\gamma_S^2 - \gamma_S^2 |\theta_S] + E[\eta_S|\theta_S] \over (N_S^L - 1)\Gamma
\]

Similarly, the optimal demand of investors who receive the signal of the long-term project is

\[
X_L^* = \frac{W_L}{\delta_L} E[\gamma_L^2 - \gamma_L^2 |\theta_L] + E[\eta_L|\theta_L] \over (N_L^L - 1)\Gamma
\]

We next need expressions for the expectations.

\[
E[\gamma_S^2 - \gamma_S^2 |\theta_S] = E[\gamma_S^2 |\theta_S] = \gamma_S^2 \epsilon_S + \eta_S + \epsilon_S - \gamma_S^2
\]

\[
= E[\gamma_S^2 |\theta_S] = \frac{W_S}{\delta_S} \gamma_S^2 \epsilon_S + \eta_S + \epsilon_S - \gamma_S^2
\]

\[
= \gamma_S^2 + \frac{\text{cov}(\gamma_S^2, W_S \gamma_S^2 \epsilon_S + \eta_S + \epsilon_S)}{\text{var}(W_S \gamma_S^2 \epsilon_S + \eta_S + \epsilon_S)} \left( \frac{W_S}{\delta_S} \gamma_S^2 \epsilon_S + \eta_S + \epsilon_S - \frac{W_S}{\delta_S} \gamma_S^2 \right) - \gamma_S^2
\]

\[
= \frac{W_S}{\delta_S} \gamma_S^2 \epsilon_S + \eta_S + \epsilon_S \left( \frac{W_S}{\delta_S} \gamma_S^2 \epsilon_S + \eta_S + \epsilon_S - \frac{W_S}{\delta_S} \gamma_S^2 \right) - \gamma_S^2
\]

\[
= \text{var}(\frac{W_S}{\delta_S} \gamma_S^2 \epsilon_S + \eta_S + \epsilon_S) \left( \frac{W_S}{\delta_S} \gamma_S^2 \epsilon_S + \eta_S + \epsilon_S - \frac{W_S}{\delta_S} \gamma_S^2 \right)
\]

\[
= \mu_S \left( \frac{W_S}{\delta_S} \Delta_S + \eta_S + \epsilon_S \right)
\]

where \(\Delta_S \equiv \gamma_S^2 - \gamma_S^2\) and \(\mu_S = \frac{W_S}{\delta_S} \sigma_S^2 \epsilon_S \). Similarly,

\[
E[\gamma_L^2 - \gamma_L^2 |\theta_L] = \mu_L \left( \frac{W_L}{\delta_L} \Delta_L + \eta_L + \epsilon_L \right)
\]

where \(\Delta_L \equiv \gamma_L^2 - \gamma_L^2\) and \(\mu_L = \frac{W_L}{\delta_L} \sigma_L^2 \epsilon_L \). We also have
Define $M \equiv \frac{W_S}{\delta S} \gamma S + \eta S$. Similarly, with $\mu_L = \frac{\sigma^2 L}{\sigma^2 L + \sigma^2 S + \sigma^2 \varepsilon}$, we have

$$E[\eta_S|\theta_S = \frac{W_S}{\delta S} \gamma S + \eta S + \varepsilon S] = \frac{\text{cov}(\eta_S, \frac{W_S}{\delta S} \gamma S + \eta S + \varepsilon S)}{\text{var}(\frac{W_S}{\delta S} \gamma S + \eta S + \varepsilon S)} \left( \frac{W_S}{\delta S} \gamma S + \eta S + \varepsilon S - \frac{W_S}{\delta S} \gamma S \right)$$

where $\mu_{\eta S} = \frac{\sigma^2 S}{\sigma^2 S + \sigma^2 S + \sigma^2 \varepsilon}$. Similarly, with $\mu_{\eta L} = \frac{\sigma^2 L}{\sigma^2 L + \sigma^2 S + \sigma^2 \varepsilon}$,

$$E[\eta_L|\theta_L = \frac{W_L}{\delta L} \gamma L + \eta L + \varepsilon L] = \mu_{\eta L} \left( \frac{W_L}{\delta L} \Delta L + \eta L + \varepsilon L \right)$$

Let $\bar{e}_S = \left( \frac{5S}{\gamma S} \right) \left( \frac{5S}{\gamma S} \right) W_S$ and $\bar{e}_L = \left( \frac{5L}{\gamma L} \right) \left( \frac{5L}{\gamma L} \right) W_L$. Then,

$$X_S^2 = \frac{\left( \frac{W_S}{\delta S} \mu S + \mu_{\eta S} \right) \left( \frac{W_S}{\delta S} \Delta S + \eta S + \varepsilon S \right)}{(N_S^2 + 1)\Gamma}$$

and

$$X_L^2 = \frac{\left( \frac{W_L}{\delta L} \mu L + \mu_{\eta L} \right) \left( \frac{W_L}{\delta L} \Delta L + \eta L + \varepsilon L \right)}{(N_L^2 + 1)\Gamma}$$

QED.

**Proof of Lemma 5: The Equilibrium Price Function**

Define $M_S \equiv \frac{W_S}{\delta S} \mu S + \mu_{\eta S}$ and $M_L \equiv \frac{W_L}{\delta L} \mu L + \mu_{\eta L}$. In equilibrium, we have

$$\Gamma = \frac{\text{Cov}(\Psi, v)}{\text{Var}(\Psi)}$$

where

$$\text{Cov}(\Psi, v) = \text{Cov}\left( \gamma S \varepsilon S + \eta S + \gamma L \varepsilon L + \eta L + \eta_L, \frac{N_S^2 M_S \left( \frac{W_S}{\delta S} \Delta S + \eta S + \varepsilon S \right)}{\left( N_S^2 + 1 \right) \Gamma} + \frac{N_L^2 M_L \left( \frac{W_L}{\delta L} \Delta L + \eta L + \varepsilon L \right)}{\left( N_L^2 + 1 \right) \Gamma} + Z \right)$$

$$= \text{Cov}\left( \frac{W_S}{\delta S} (\gamma S^2 + \Delta S) + \eta S + \frac{W_L}{\delta L} (\gamma L^2 + \Delta L) + \eta L + \eta_L, \frac{N_S^2 M_S \left( \frac{W_S}{\delta S} \Delta S + \eta S + \varepsilon S \right)}{\left( N_S^2 + 1 \right) \Gamma} + \frac{N_L^2 M_L \left( \frac{W_L}{\delta L} \Delta L + \eta L + \varepsilon L \right)}{\left( N_L^2 + 1 \right) \Gamma} + Z \right)$$

$$= \left( \frac{N_S^2 M_S}{\left( N_S^2 + 1 \right) \Gamma} \right) \left[ \frac{W_S}{\delta S} \gamma S^2 + \sigma^2 \varepsilon \right] + \left( \frac{N_L^2 M_L}{\left( N_L^2 + 1 \right) \Gamma} \right) \left[ \frac{W_L}{\delta L} \gamma L^2 + \sigma^2 \varepsilon \right]$$
Since and Recall that the price function is

Proof of Lemma 6: Expected Residual Value

Thus, the price function can be written as

Recall that \( \Gamma = \left( \gamma_S^2 + \Delta_S \right) \) and \( \gamma_L \in \left( \gamma_L^2 + \Delta_L \right) \). We can solve for \( \Gamma \) in the following equation

which implies

Recall that \( p = p_0 + \lambda \Psi \). Thus,

Since \( p = p_0 + \lambda \Psi = p_0 + \left( \frac{1 - \omega_i}{1 + \omega_i} \right) \Gamma \left[ \pi_S (\theta_S - E[\theta_S]) + \pi_L (\theta_L - E[\theta_L]) + Z \right] \) and \( \theta_i - E[\theta_i] = \gamma_i (e_i - \bar{e}_i) + \eta_i + \varepsilon_i \), we have

and

Recall that \( W_i = \omega_p \pi_i \lambda + \omega_\nu \). Thus, the coefficients in the price function \( \pi_S \) and \( \pi_L \) are given by the following (for \( i = S, L \)):

Thus, the price function can be written as

where \( \eta_i \equiv \lambda_i \bar{\eta}_i \) and \( \varepsilon_i \equiv \lambda_i \bar{\varepsilon}_i \), and \( z = \left( \frac{1 - \omega_i}{1 + \omega_i} \right) \Gamma Z \). QED.

**Proof of Lemma 6: Expected Residual Value**

Recall that the price function is

\[ p = p_0 + \lambda_S \gamma_S (e - \bar{e}_S) + \lambda_L \gamma_L (e - \bar{e}_L) + \bar{\eta}_S + \bar{\varepsilon}_S + \bar{\eta}_L + \bar{\varepsilon}_L + z \]
Note that the unconditional expectation of \( p \) is \( E[p] = p_0 \). The expected utility of the agent is

\[
E[u] = \omega_0 + \omega_p\left[ p_0 + \sum_{i=S, L} \lambda_i \gamma_i (e - \bar{e}_i) \right] + \omega_v \left[ \sum_{i \in S, L} \gamma_i e_i \right] - \frac{\gamma}{2} \text{var}(w) + E \left[ \sum_{i \in S, L} \frac{\delta_i}{2} e_i^2 \right]
\]

where

\[
\text{var}(w) = \text{var}(\omega_0 + \omega_p p + \omega_v v) = \text{var}\left( \omega_p [ p_0 + \lambda_S \gamma_S (eS - \bar{e}_S) + \lambda_L \gamma_L (eL - \bar{e}_L) + \lambda_S \eta_S + \lambda_L \eta_L + \lambda_L \bar{e}_L + \bar{e}_L] \right) + \omega_v \left[ \gamma_S \epsilon_S + \gamma_L \epsilon_L + \eta_S + \eta_L \right]
\]

\[
= \text{var}\left( \omega_p [ \lambda_S \eta_S + \lambda_S \epsilon_S + \lambda_L \eta_L + \lambda_L \bar{e}_L + \bar{e}_L] \right) + \omega_v \left[ \gamma_S \epsilon_S + \gamma_L \epsilon_L + \eta_S + \eta_L \right]
\]

\[
= \text{var}\left( \omega_p [ \lambda_S \eta_S + \lambda_S \epsilon_S + \lambda_L \eta_L + \lambda_L \bar{e}_L + \bar{e}_L] \right)
\]

Substituting for \( e_i = \bar{e}_i = \frac{\omega_p}{\delta} W_i = \frac{\omega_p}{\delta} [\omega_p \lambda_i + \omega_v] \) and taking expectations yields

\[
E[u] = \omega_0 + \omega_p p_0 + \omega_v \left[ \sum_{i \in S, L} \frac{\kappa_i^2}{\delta_i} W_i \right] - \frac{\gamma}{2} \left[ (W_S)^2 \sigma_{\eta_S}^2 + (\omega_p \lambda_S)^2 \sigma_{\epsilon_S}^2 + (W_L)^2 \sigma_{\eta_L}^2 + (\omega_p \lambda_L)^2 \sigma_{\epsilon_L}^2 + (\omega_v)^2 \sigma_{\eta_T}^2 \right]
\]

Substitute \( p_0 = \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left( \frac{\kappa_{\delta S}}{\delta} W_S + \frac{\kappa_{\delta L}}{\delta} W_L \right) \) (from Lemma 5) and set to zero to solve for \( \omega_0 \):

\[
0 = E[u] = \omega_0 + \omega_p \left[ \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left( \frac{\kappa_{\delta S}}{\delta} W_S + \frac{\kappa_{\delta L}}{\delta} W_L \right) - \frac{\omega_0}{1 + \omega_p} \right] + \omega_v \sum_{i \in S, L} \frac{\kappa_i^2}{\delta_i} W_i
\]

\[
- \frac{\gamma}{2} \left[ (W_S)^2 \sigma_{\eta_S}^2 + (\omega_p \lambda_S)^2 \sigma_{\epsilon_S}^2 + (W_L)^2 \sigma_{\eta_L}^2 + (\omega_p \lambda_L)^2 \sigma_{\epsilon_L}^2 + (\omega_v)^2 \sigma_{\eta_T}^2 \right] - \frac{1}{2} \sum_{i \in S, L} \frac{\kappa_i^2}{\delta_i} (W_i)^2
\]

\[
\Rightarrow \omega_0 \left[ 1 - \frac{\omega_0}{1 + \omega_p} \right] = \frac{\gamma}{2} \left[ (W_L)^2 \sigma_{\eta_L}^2 + (\omega_p \lambda_L)^2 \sigma_{\epsilon_L}^2 + (W_S)^2 \sigma_{\eta_S}^2 + (\omega_p \lambda_S)^2 \sigma_{\epsilon_S}^2 + (\omega_v)^2 \sigma_{\eta_T}^2 \right]
\]

\[
+ \frac{1}{2} \sum_{i \in S, L} \frac{\kappa_i^2}{\delta_i} (W_i)^2 - \omega_p \left[ \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left( \frac{\kappa_{\delta S}}{\delta} W_S + \frac{\kappa_{\delta L}}{\delta} W_L \right) \right] - \omega_v \sum_{i \in S, L} \frac{\kappa_i^2}{\delta_i} W_i
\]

\[
\Rightarrow \omega_0 = (1 + \omega_p) \left\{ \frac{\gamma}{2} \left[ (W_L)^2 \sigma_{\eta_L}^2 + (\omega_p \lambda_L)^2 \sigma_{\epsilon_L}^2 + (W_S)^2 \sigma_{\eta_S}^2 + (\omega_p \lambda_S)^2 \sigma_{\epsilon_S}^2 + (\omega_v)^2 \sigma_{\eta_T}^2 \right]
\]

\[
+ \frac{1}{2} \sum_{i \in S, L} \frac{\kappa_i^2}{\delta_i} (W_i)^2 - \omega_p \left[ \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left( \frac{\kappa_{\delta S}}{\delta} W_S + \frac{\kappa_{\delta L}}{\delta} W_L \right) \right] - \omega_v \sum_{i \in S, L} \frac{\kappa_i^2}{\delta_i} W_i \right\}
\]

Moving to the principals problem, we have

\[
\max_{\omega_p, \omega_v} E[v] - E[u]
\]

We have

\[
E[v] = \frac{\kappa_{\delta S}}{\delta} W_S + \frac{\kappa_{\delta L}}{\delta} W_L
\]

48
\[ E[w] = \omega_0^* + \omega_p p_0 + \omega_v \left( \frac{\gamma_3}{\delta_S} W_S + \frac{\gamma_2}{\delta_L} W_L \right) \]

\[ = \omega_0^* + \omega_p \left[ \frac{1 - \omega_v}{1 + \omega_p} \right] \left( \frac{\gamma_3}{\delta_S} W_S + \frac{\gamma_2}{\delta_L} W_L \right) + \omega_v \left( \frac{\gamma_2}{\delta_S} W_S + \frac{\gamma_2}{\delta_L} W_L \right) \]

\[ = \omega_0^* + \omega_p \left[ \frac{1 - \omega_v}{1 + \omega_p} \right] \left( \frac{\gamma_2}{\delta_S} W_S + \frac{\gamma_2}{\delta_L} W_L \right) \]

\[ = \left\{ \frac{\gamma}{2} \left[ (W_S)^2 \sigma_{\pi_S}^2 + (\omega_p \lambda_S)^2 \sigma_{\pi_S}^2 + (W_L)^2 \sigma_{\pi_L}^2 + (\omega_p \lambda_L)^2 \sigma_{\pi_L}^2 + (\omega_v \lambda)^2 \sigma_{\pi_V}^2 \right] \right. \]

\[ + \frac{1}{2} \frac{\gamma}{\delta} \sum_{i=S,L} \left[ (W_i)^2 - \omega_p \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left( \frac{\gamma_2}{\delta_S} W_S + \frac{\gamma_2}{\delta_L} W_L \right) \right] - \omega_v \sum_{i=S,L} \frac{\gamma_2}{\delta_i} W_i \]

\[ = \frac{\gamma}{2} \left[ (W_S)^2 \sigma_{\pi_S}^2 + (\omega_p \lambda_S)^2 \sigma_{\pi_S}^2 + (W_L)^2 \sigma_{\pi_L}^2 + (\omega_p \lambda_L)^2 \sigma_{\pi_L}^2 + (\omega_v \lambda)^2 \sigma_{\pi_V}^2 \right] + \sum_{i=S,L} \frac{\gamma_2}{\delta_i} (W_i)^2 \]

Thus, the objective function for the shareholders is

\[ E[v] - E[w] = \frac{\gamma}{2} \left[ (W_S)^2 \sigma_{\pi_S}^2 + (\omega_p \lambda_S)^2 \sigma_{\pi_S}^2 + (W_L)^2 \sigma_{\pi_L}^2 + (\omega_p \lambda_L)^2 \sigma_{\pi_L}^2 + (\omega_v \lambda)^2 \sigma_{\pi_V}^2 \right] \]

\[ + \frac{1}{2} \frac{\gamma}{\delta} \sum_{i=S,L} \left[ (W_i)^2 - \omega_p \left( \frac{1 - \omega_v}{1 + \omega_p} \right) \left( \frac{\gamma_2}{\delta_S} W_S + \frac{\gamma_2}{\delta_L} W_L \right) \right] - \omega_v \sum_{i=S,L} \frac{\gamma_2}{\delta_i} W_i \]

where

\[ \lambda_i = \frac{\left( \frac{N_i}{N + 1} \right) \left( \omega_p \mu_i + \mu_{\pi} \right)}{\left( \frac{1 + \omega_p}{1 + \omega_v} \right) - \left( \frac{N_i}{N + 1} \right) \left( \frac{\gamma_2}{\delta_S} \mu_i \right)} \]

QED.

**Proof of Lemma 7: Expected Trade Profits**

The expected profit of an investor receiving a signal of the short term project is defined as \( E[X_{m}^S (v' - p) | \theta_S] \). From page 43 we have,

\[ E \left[ X_{m}^S (v' - p) | \theta_S \right] = (1 - \omega_v) E \left[ X_{m}^S \{(v - E(v)) - \Gamma \Psi\} | \theta_S \right] \]

From page 45,

\[ = (1 - \omega_v) X_{m}^S \left[ \frac{W_S}{\delta_S} E \left[ \gamma_3^2 - \gamma_3^2 | \theta_S \right] + E [\eta_S | \theta_S] \right] - \Gamma \left( X_{m}^S + (N_d^S - 1) E[X_{m}^S | \theta_S] \right) \]

where \( X_{m}^S \) denotes the trades of the other investors with a type-S signal. Let \( \alpha \equiv E[\gamma_3^2 - \gamma_3^2 | \theta_S] \) and \( \beta \equiv E[\eta_S | \theta_S] \) and rewrite as

\[ E \left[ X_{m}^S (v' - p) | \theta_S \right] = (1 - \omega_v) X_{m}^S \left[ \frac{W_S}{\delta_S} \alpha + \beta - \Gamma X_{m}^S N_d^S \right] \]

From page 45 we have the optimal demand of an investor with signal type-S equal to

\[ X_{m}^S = \frac{W_S}{\Gamma(N_d^S + 1)} \]

This implies
\[
E\left[X_m^{S}(v'-p)|\theta_S\right] = (1-\omega_v)(W_s\frac{\alpha + \beta}{\delta_S})^2 - \Gamma N_S^2 \left(\frac{W_s}{\Gamma(N_S + 1)}\right)^2
\]
\[
= (1-\omega_v)(W_s\frac{\alpha + \beta}{\delta_S})^2
\]

Now substitute \(\alpha\) and \(\beta\) from previous results: \(\alpha = \mu_S\left(\frac{W_s}{\delta_S} \Delta_S + \eta_S + \varepsilon_S\right), \text{ and } \beta = \mu_{\eta_S}\left(\frac{W_s}{\delta_S} \Delta_S + \eta_S + \varepsilon_S\right). \) Thus,

\[
E\left[X_m^{S}(v'-p)|\theta_S\right] = (1-\omega_v)\left(W_s\frac{\alpha + \beta}{\delta_S}\right)^2 \Gamma(N_S + 1)^2
\]

Similarly the expected profit of an investor receiving a signal of the long term project is:

\[
E\left[X_m^{L}(v'-P)|\theta_L\right] = (1-\omega_v)\left(W_L\frac{\mu_L + \mu_{\eta_L}}{\delta_L}\Delta_L + \eta_L + \varepsilon_L\right)^2 \Gamma(N_L + 1)^2
\]

To calculate the unconditional expected profit, we use the identity \(E[E[X|Y]] = E[X].\) Taking the expectation of the results above, \(E[\Pi_i|\theta_i] = E[\Pi_i]\) where \(\Pi_i\) is the profit from receiving a short signal.

\[
= E\left\{ (1-\omega_v)\left(W_s\frac{\alpha + \beta}{\delta_S}\right)^2 \Gamma(N_S + 1)^2 \right\}
\]

\[
= (1-\omega_v)\left(W_s\frac{\alpha + \beta}{\delta_S}\right)^2 \Gamma(N_S + 1)^2
\]

Recall that \(\Delta_S = \gamma_S^2 - \eta_S^2, \text{ and } \Delta_S, \eta_S, \varepsilon_S\) are all uncorrelated. In the expectation, after multiplying the variables in parenthesis by itself, the only thing that survives the expectation is the square terms. That is

\[
E\left[\left(\frac{W_s}{\delta_S} \Delta_S + \eta_S + \varepsilon_S\right)^2\right] = \left(\frac{W_s}{\delta_S}\right)^2 E[\Delta_S]^2 + E[\eta_S]^2 + E[\varepsilon_S]^2
\]

And we know that \(\text{Var}[X] = E[X^2] - E[X]^2\) since each as a mean zero,

\[
\left(\frac{W_s}{\delta_S}\right)^2 E[\Delta_S]^2 + E[\eta_S]^2 + E[\varepsilon_S]^2 = \left(\frac{W_s}{\delta_S}\right)^2 \sigma_\Delta^2 + \sigma_\eta^2 + \sigma_\varepsilon^2
\]

Substitute this back into \(A.6\) yielding

\[
E[\Pi_S] = \frac{(1-\omega_v)(W_s\frac{\alpha + \beta}{\delta_S})^2}{\Gamma(N_S + 1)^2} \left[\left(\frac{W_s}{\delta_S}\right)^2 \sigma_\Delta^2 + \sigma_\eta^2 + \sigma_\varepsilon^2\right]
\]

Similarly,

\[
E[\Pi_L] = \frac{(1-\omega_v)(W_L\frac{\mu_L + \mu_{\eta_L}}{\delta_L})^2}{\Gamma(N_L + 1)^2} \left[\left(\frac{W_L}{\delta_L}\right)^2 \sigma_\Delta^2 + \sigma_\eta^2 + \sigma_\varepsilon^2\right]
\]

QED.

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Table I. Time Line of the Model

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>The board of directors offers the manager a contract ((ω₀, ωₚ, ωᵥ)).</td>
<td>Investors (or speculators) observe costly noisy signals (θ_S(θ_L)) of the value of short-term (long-term) project.</td>
<td>The terminal value of the firm (v) is realized.</td>
</tr>
<tr>
<td></td>
<td>The short-term (v_S), and the long-term (v_L) projects are initiated by the manager.</td>
<td>The market for the firm’s shares opens for trade; and the resulting market price is (p).</td>
<td>The manager is (partially) compensated an amount of (ωᵥp).</td>
</tr>
<tr>
<td></td>
<td>The initial price of the stock (p₀) (unconditional expectation of the net terminal value) is determined by the investors</td>
<td>The manager is (partially) compensated an amount of (ω₀v).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>The manager is (partially) compensated a fixed wage of (ω₀).</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

0 1 2
• The board of directors offers the manager a contract \((ω₀, ωₚ, ωᵥ)\).
• The short-term \(v_S\), and the long-term \(v_L\) projects are initiated by the manager.
• The initial price of the stock \(p₀\) (unconditional expectation of the net terminal value) is determined by the investors.
• The manager is (partially) compensated a fixed wage of \(ω₀\).
Figure 1. Level Sets for Expected Residual Value given Combinations of $\lambda_S$ and $\lambda_L$. 

\[ \begin{array}{c}
\lambda_S \\
\lambda_L
\end{array} \]
Figure 2. Market Fails to Achieve the Maximal Expected Residual Value
Figure 3. Effect of Changing Conditions (Case 1)
Figure 4. Effect of Changing Conditions (Case 2)

\[ E(\Pi|(\omega_{p0}, \omega_{v0}), \Omega_0) = 0 \]

\[ E(\Pi|(\omega_{p0}, \omega_{v0}), \Omega_1) = 0 \]
Figure 5. Fixity in Total Number of Speculator and the Achievement of Maximal Expected Residual Value
Figure 6. Less Fixity in the Total Number of Speculators than the Contract
Figure 7. Either a Short-term Tax or a Long-Term Subsidy Increases Expected Residual Value
Figure 8. A Tax on Short-Term Speculation Lower Expected Residual Value

\[ E(\Pi| (\omega_{p0}, \omega_{r0}), \Omega_0, T_{S} = T_{L} = S_S = S_L = 0) = 0 \]

\[ E(\Pi| (\omega_{p0}, \omega_{r0}), \Omega_0, T_{S} = T_{L} = S_S = 0, S_L > 0) = 0 \]

\[ E(\Pi| (\omega_{p0}, \omega_{r0}), \Omega_0, T_S > 0, T_L = S_S = S_L = 0) = 0 \]